SAT-based Model Checking for C programs

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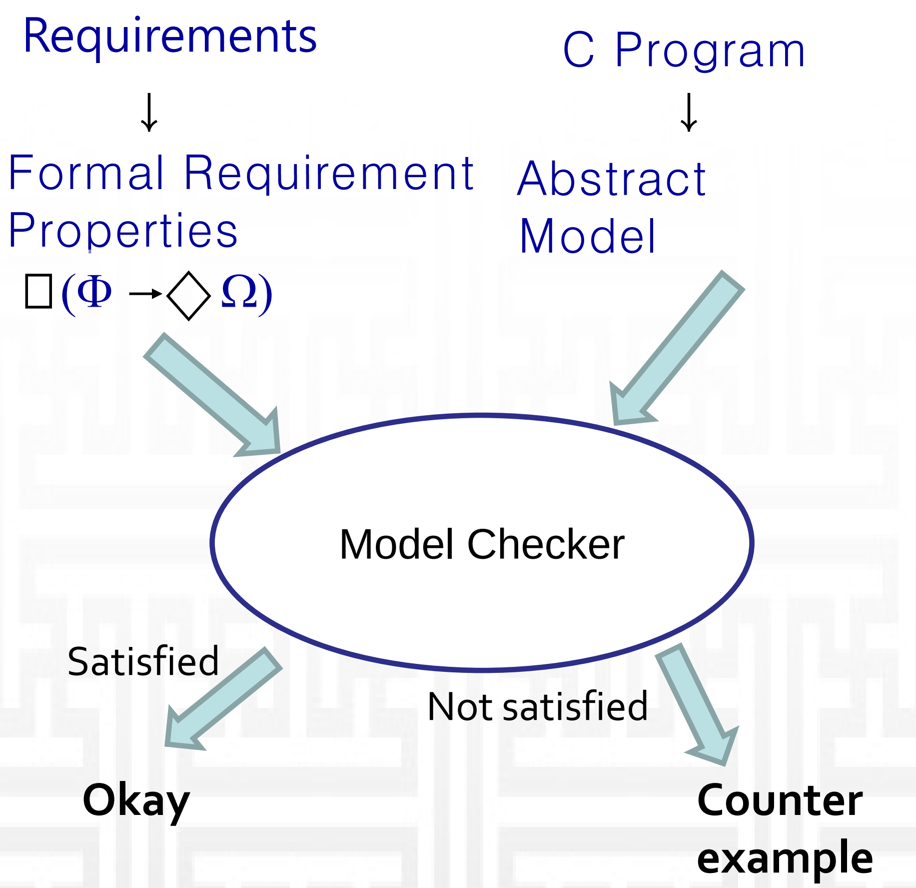
Formal Methods

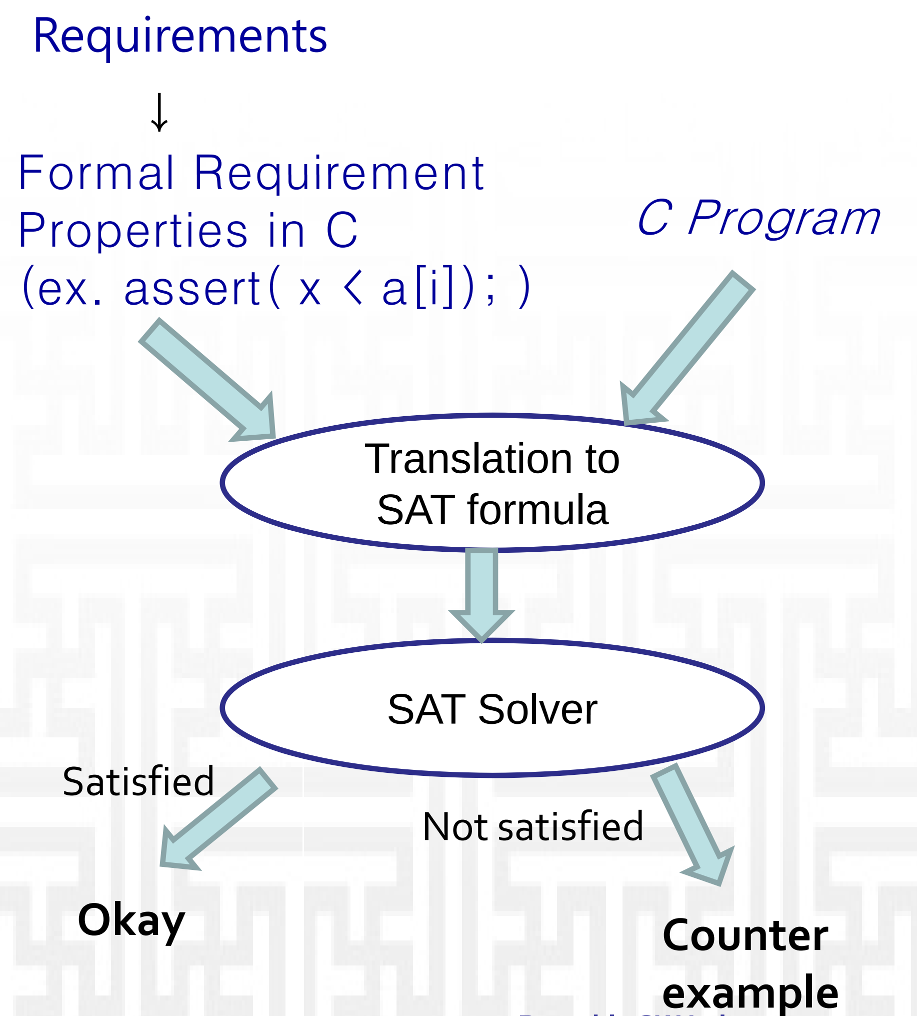
- **Definition in Wikepedia**
  - "**Formal methods** are mathematically-based techniques for the specification, development and verification of software … performing appropriate mathematical analyses can contribute to the reliability and robustness of a design."
    - Rigor, completeness
    - Automation
    - Scalability

    ⟷ Ad-hoc manual development

- **Brief History**
  - Early days: 1960~ :Theorem proving
    - Hoare logic – pre & post condition, loop invariant
    - Heavily depends on human expertise
  - Mid days: 1980~: Model checking
    - Design an abstract model and check the model with requirements
    - Fully automatic analysis
  - Recent days: 2000~: Software model checking
    - (semi) automatic model extraction from C code
    - Not yet reliable and weak tool supports

Overview of SAT-based Bounded Model Checking





SAT Basics (1/2)

- **SAT = Satisfiability**
  = Propositional Satisfiability
- **NP-Complete problem**
  - We can use SAT solver for many NP-complete problems
    - Hamiltonian path
    - 3 coloring problem
    - Traveling sales man's problem
- **Recent interest as a verification engine**

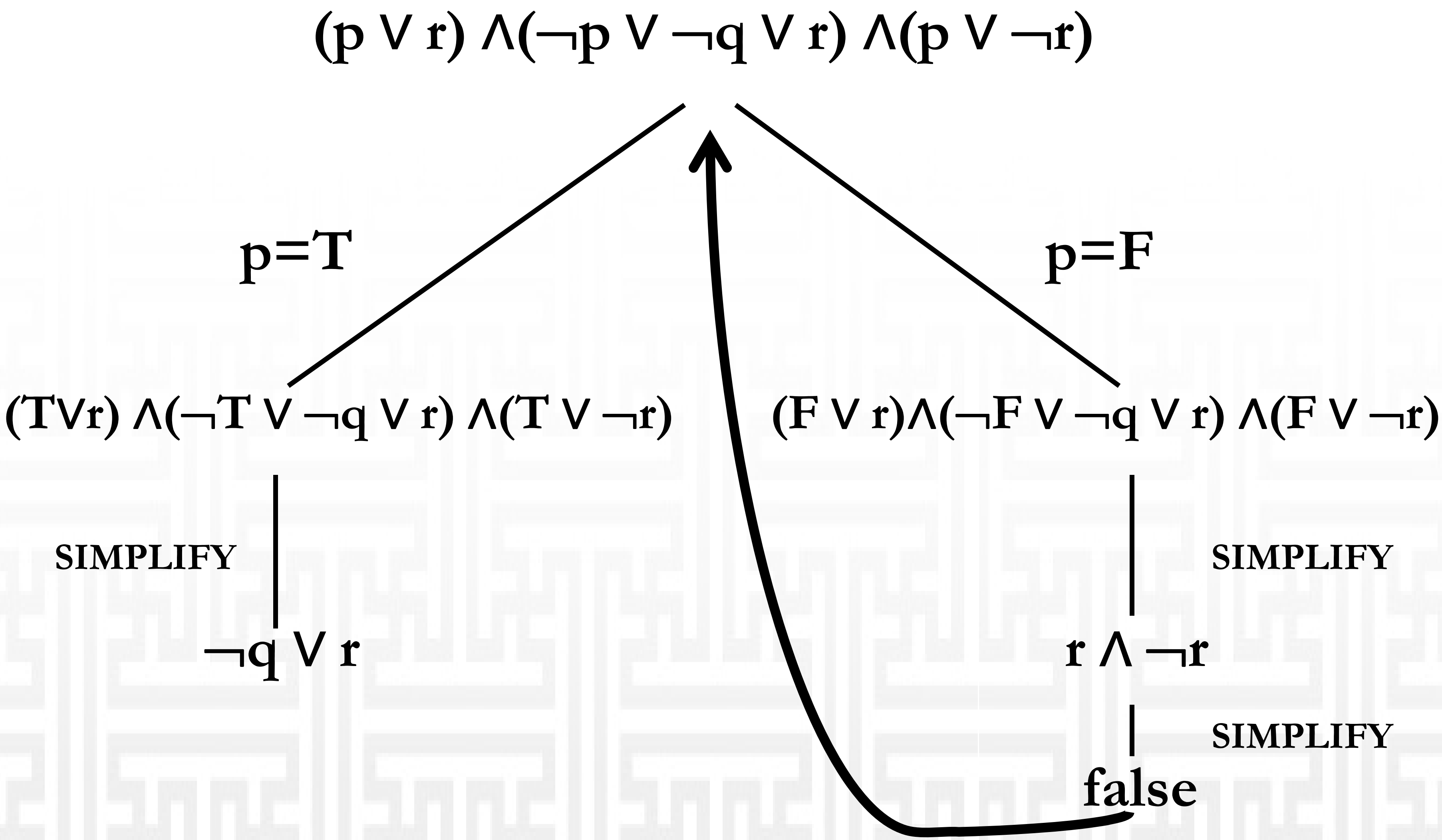
SAT Basics (2/2)

- **A set of propositional variables and clauses involving variables**
  - $(x_1 \vee x_2' \vee x_3) \wedge (x_2 \vee x_1' \vee x_4)$
  - x_1, x_2, x_3 and x_4 are variables (true or false)
- **Literals: Variable and its negation**
  - x_1 and x_1'
- **A CNF clause is satisfied if one of the literals is true**
  - x_1=true satisfies clause 1
  - x_1=false satisfies clause 2
- **Solution: An assignment that satisfies all clauses**

Basic SAT Solving Mechanism (1/2)

```
/* The Quest for Efficient Boolean Satisfiability Solvers
 *  by L.Zhang and S.Malik, Computer Aided Verification 2002 */
DPLL(a formula φ, assignment) {
    necessary = deduction(φ, assignment);
    new_asgnment = union(necessary, assignment);
    if (is_satisfied(φ, new_asgnment))
            return SATISFIABLE;
    else if (is_conflicting(φ, new_asgnmnt))
            return UNSATISFIABLE;
    var = choose_free_variable(φ, new_asgnmnt);
    asgn1 = union(new_asgnmnt, assign(var, 1));
    if (DPLL(φ, asgn1) == SATISFIABLE)
            return SATISFIABLE;
    else {
            asgn2 = union (new_asgnmnt, assign(var,0));
            return DPLL (φ, asgn2);
    }
}
```

Basic SAT Solving Mechanism (2/2)



Model Checking as a SAT problem (1/4)

- **CBMC (C Bounded Model Checker, In CMU)**
  - Handles function calls using inlining
  - Unwinds the loops a fixed number of times (bounded MC)
    - Thus, a user has to know a upper bound of each loop
      - In practice, it does not cause serious limitation since
        - » Most loops has clear upper bounds
        - » Even when we do not know the upper bound, we can still get analysis result with possibility of false alarms
  - Allows user input to be modeled using non-determinism
    - So that a program can be checked for a set of inputs rather than a single input
  - Allows specification of assertions which are checked using the bounded model checking

Model Checking as a SAT problem (2/4)

- **Unwinding Loop**

Original code

```
x=0;
while (x < 2) {
  y=y+x;
  x++;
}
```

Unwinding the loop 3 times

```
x=0;
if (x < 2) {
  y=y+x;
  x++;
}
if (x < 2) {
  y=y+x;
  x++;
}
if (x < 2) {
  y=y+x;
  x++;
}
```

Unwinding assertion: →

```
assert (! (x < 2))
```

Model Checking as a SAT problem (3/4)

- **From C Code to SAT Formula**

Original code

```
x=x+y;
if (x!=1)
  x=2;
else
  x++;
assert(x<=3);
```

Convert to static single assignment

$x_1 = x_0 + y_0;$
if $(x_1 != 1)$
  $x_2 = 2;$
else
  $x_3 = x_1 + 1;$
$x_4 = (x_1 != 1) ? x_2 : x_3;$
assert$(x_4 <= 3);$

Generate constraints

$C \equiv x_1 = x_0 + y_0 \wedge x_2 = 2 \wedge x_3 = x_1 + 1 \wedge (x_1 != 1 \wedge x_4 = x_2 \vee x_1 = 1 \wedge x_4 = x_3)$

$P \equiv x_4 <= 3$

Check if $C \wedge \neg P$ is satisfiable, if it is then the assertion is violated

$C \wedge \neg P$ is converted to Boolean logic using a bit vector representation for the integer variables $y_0, x_0, x_1, x_2, x_3, x_4$

Model Checking as a SAT problem (4/4)

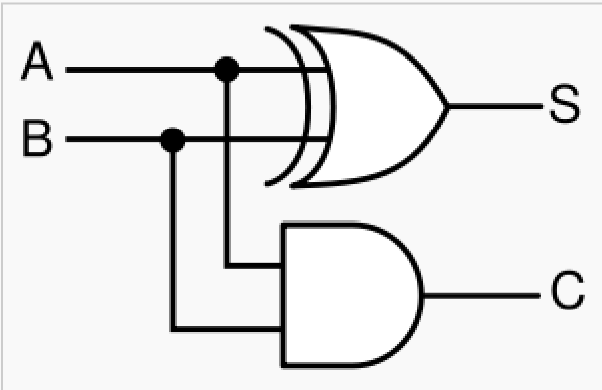
- Example of arithmetic encoding into pure propositional formula

  Assume that x,y,z are three bits positive integers represented by propositions $x_0x_1x_2$, $y_0y_1y_2$, $z_0z_1z_2$

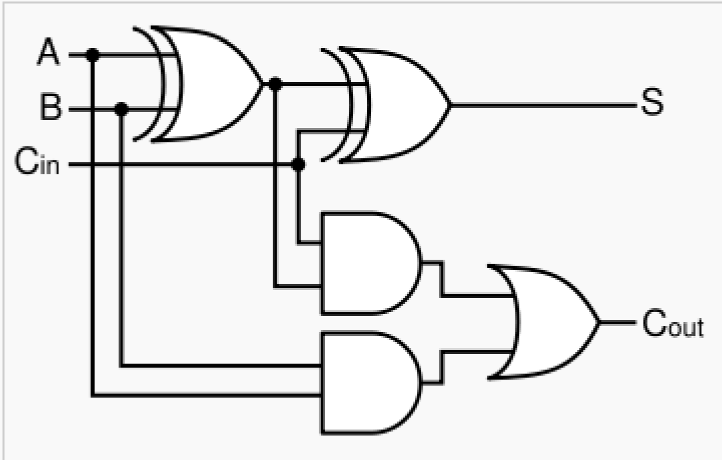
  $C \equiv z=x+y \equiv \ (z_0 \leftrightarrow (x_0 \oplus y_0) \oplus ((x_1 \wedge y_1) \vee (((x_1 \oplus y_1) \wedge (x_2 \wedge y_2)))$

  $\wedge\ (z_1 \leftrightarrow (x_1 \oplus y_1) \oplus (x_2 \wedge y_2))$

  $\wedge\ (z_2 \leftrightarrow (x_2 \oplus y_2))$



Half adder circuit diagram

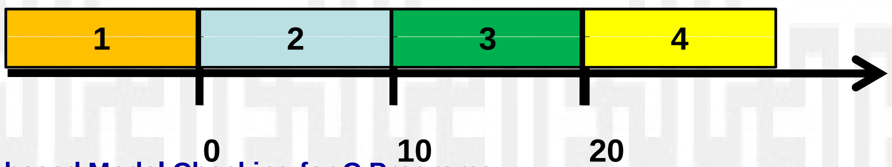


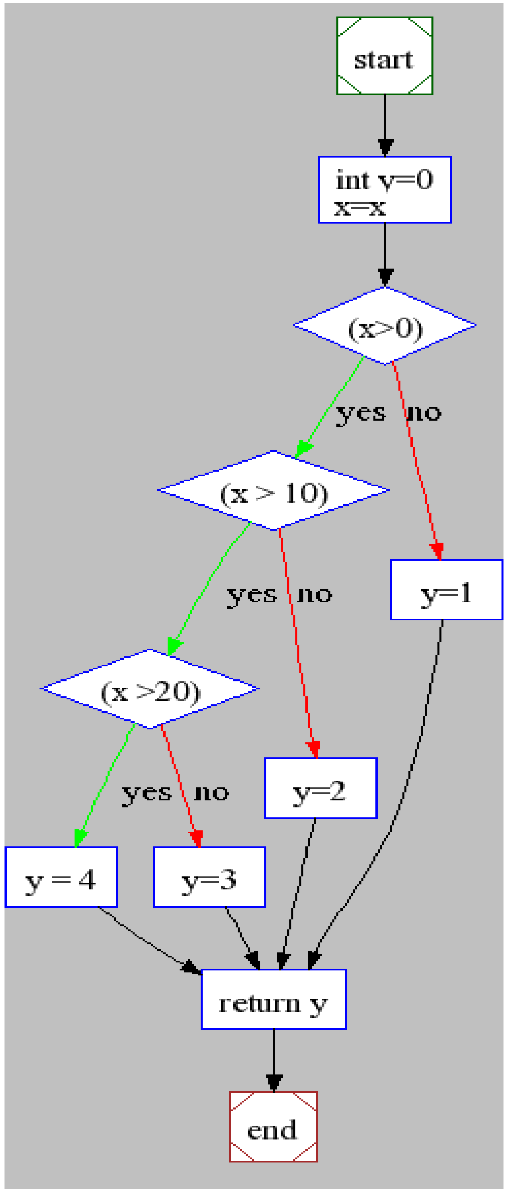
Full adder circuit diagram

Inputs: {A, B, CarryIn} → Outputs: {Sum, CarryOut}

Model Checking Example (1/3)

```
#include<stdio.h>

int many_branch(int x) {
    int y=0;
    /* Required to show meaningful counter
     * example for non-deterministic value*/
    x=x;

    if( x>0 )
        if ( x > 10)
            if (x >20)
                y = 4;
            else y=3;
        else y=2;
    else y=1;

#ifdef verification
    assert( 2<= y && y <= 4);
#endif
    return y;
}
```



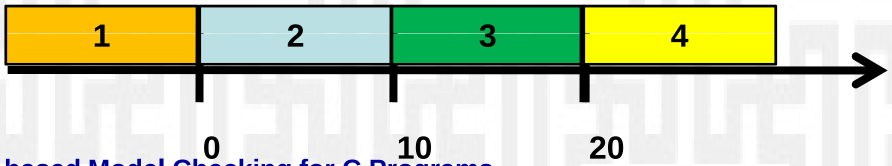


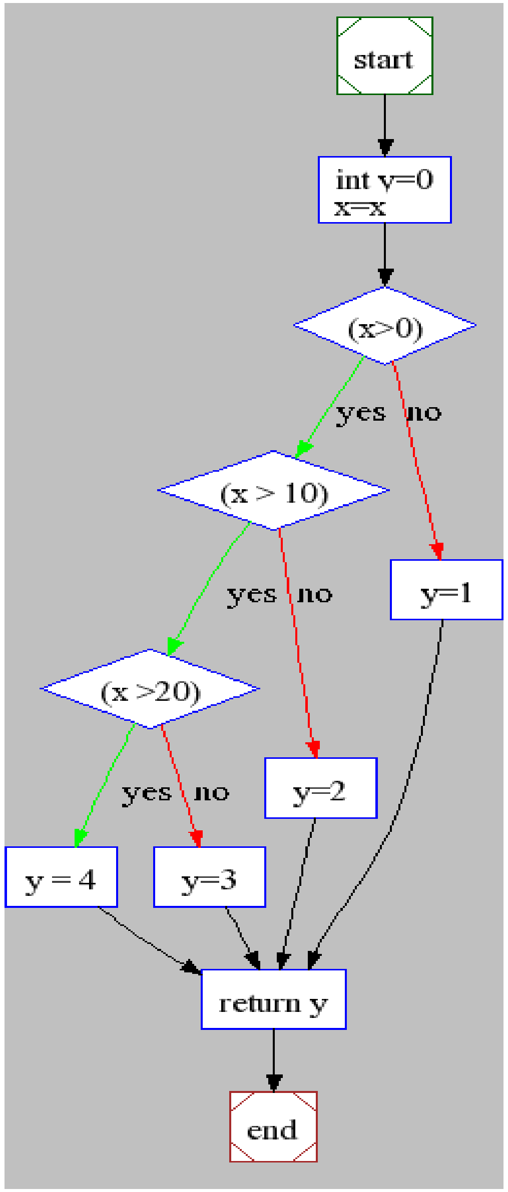
Cyclomatic complexity = 4

Model Checking Example (2/3)

```
int main() {
    int input, output;
    printf("Input x:");
    scanf("%d", &input);
    output=many_branch(input);
    printf("Output y:%d\n",output);
}
```

```
[moonzoo@verifier cs350]$  a.out
Input x:-5
Output y:1
[moonzoo@verifier cs350]$  a.out
Input x:5
Output y:2
[moonzoo@verifier cs350]$  a.out
Input x:15
Output y:3
[moonzoo@verifier cs350]$  a.out
Input x:25
Output y:4
```



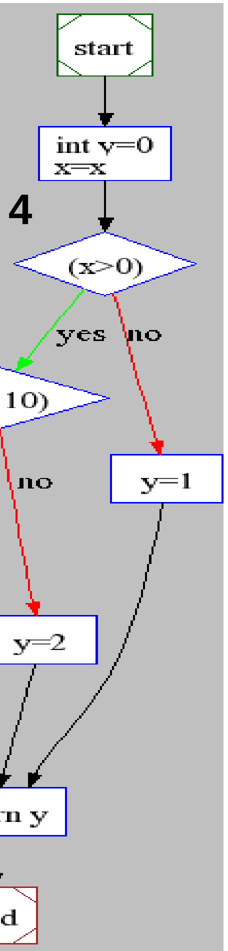


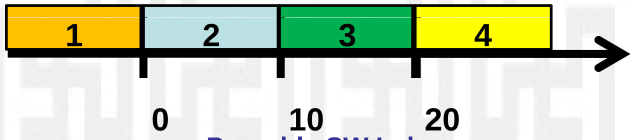
Cyclomatic complexity = 4

Model Checking Example (3/3)

```
[moonzoo@verifier cs350]$ cbmc -D verification --function many_branch test.c
Starting Bounded Model Checking
size of program expression: 30 assignments
388 variables, 897 clauses
SAT checker: negated claim is SATISFIABLE, i.e., does not hold
Building error trace

Counterexample:

Initial State file /usr/include/bits/sys_errlist.h line 27 thread 0
----------------------------------------------------
  sys_nerr=0

State 2 file test-all-branch.c line 4 function many_branch thread 0
----------------------------------------------------
  test-all-branch::many_branch::1::y=0

State 3 file test-all-branch.c line 7 function many_branch thread 0
----------------------------------------------------
  test-all-branch::many_branch::x=-2147483617

State 5 file test-all-branch.c line 15 function many_branch thread 0
----------------------------------------------------
  test-all-branch::many_branch::1::y=1

Violated property:
  file test-all-branch.c line 18 function many_branch
  assertion
  2 <= y && y <= 4
VERIFICATION FAILED
```

Cyclomatic complexity = 4





Model Checking Performances: Sort (1/2)

- **Suppose that we have an array of 4 elements each of which is 32 bits long**
  - int a[4];

9	14	2	999

- **We wants to verify sort.c works correctly**
  - main() { int a[4]; sort(); assert(a[0]<= a[1]<= a[2]<=a[3]);}
  - Note that local variables are initialized with non-deterministic values.
- **The # of all possible test cases is $5.4\text{x}10^{39}$ ($=2^{32\text{x}4}$)**
  - Exhaustive testing will takes $1.8\text{x}10^{32}$ seconds (= $5.8 \text{ x } 10^{24}$ years)
    - $5.4\text{x}10^{39}$ cases x 100 instruction/case x $1/(3\text{x}10^{9})$ sec/instruction

Model Checking Performances: Sort (2/2)

```
#include <stdio.h>
#define N 5
int main(){
  int data[N], i, j, tmp;

  /* if a function body is not provided,
   * function invocation of the function is ignored */
  for (i=0; i<N; i++) {
      data[i] = non_det();
      data[i]=data[i];
  }

  /* It misses the last element, i.e., data[N0]*/
  for (i=0; i<N-1; i++)
      for (j=i+1; j<N-1; j++)
          if (data[i] > data[j]){
              tmp = data[i];
              data[i] = data[j];
              data[j] = tmp;
          }
/* Check the array is sorted */
  for (i=0; i<N-1; i++)
      assert(data[i] <= data[i+1]);
}
```

- Total 6224 CNF clause with 19099 boolean propositional variables
- Theoretically, 2^{19099} (2.35×10^{5749}) choices should be evaluated!!!

SAT	
Conflicts	73
Decisions	2435
Time(sec)	0.015

UNSAT	
Conflicts	35067
Decisions	161406
Time(sec)	1.89

Excerpts from Microsoft Research Conf. '06