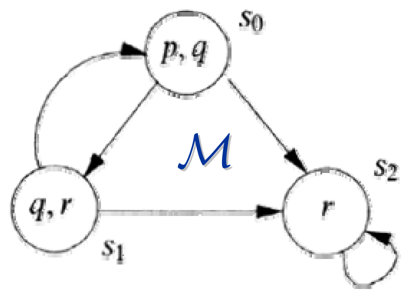
Temporal Logic (2/2)

Moonzoo Kim
CS Division of EECS Dept.
KAIST

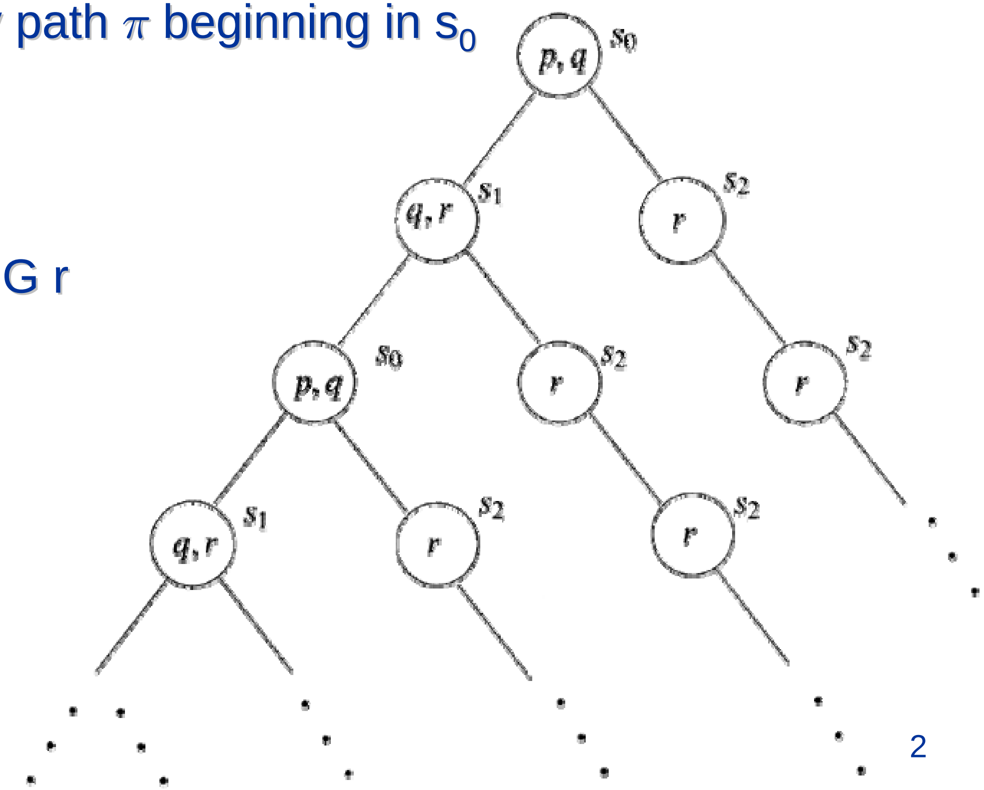
moonzoo@cs.kaist.ac.kr
http://pswlab.kaist.ac.kr/courses/cs402-07

Semantics of LTL (3/3)

- Def 3.8 Suppose $\mathcal{M} = (S, \rightarrow, L)$ is a model, $s \in S$, and ϕ an LTL formula. We write $\mathcal{M}, s \vDash \phi$ if for every execution path π of $\mathcal{M}$ starting at s, we have $\pi \vDash \phi$
  - If $\mathcal{M}$ is clear from the context, we write $s \vDash \phi$



- Example
  - $s_0 \vDash p \wedge q$ since $\pi \vDash p \wedge q$ for every path π beginning in s_0
  - $s_0 \vDash \neg r$, $s_0 \vDash \top$
  - $s_0 \vDash X\, r$, $s_0 \nvDash X\, (q \wedge r)$
  - $s_0 \vDash G\, \neg(p \wedge r)$, $s_2 \vDash G\, r$
  - For any s of $\mathcal{M}$, $s \vDash F(\neg q \wedge r) \rightarrow F\, G\, r$
    - Note that s_2 satisfies $\neg q \wedge r$
  - $s_0 \nvDash G\, F\, p$
    - $s_0 \rightarrow s_1 \rightarrow s_0 \rightarrow s_1 \ldots \vDash G\, F\, p$
    - $s_0 \rightarrow s_2 \rightarrow s_2 \rightarrow s_2 \ldots \nvDash G\, F\, p$
  - $s_0 \vDash G\, F\, p \rightarrow G\, F\, r$
  - $s_0 \nvDash G\, F\, r \rightarrow G\, F\, p$



Practical patterns of specification

- For any state, if a request occurs, then it will eventually be acknowledge
  - G(requested $\rightarrow$ F acknowledged)
- A certain process is enabled infinitely often on every computation path
  - G F enabled
- Whatever happens, a certain process will eventually be permanently deadlocked
  - F G deadlock
- If the process is enabled infinitely often, then it runs infinitely often
  - G F enabled $\rightarrow$ G F running
- An upwards traveling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor
  - G (fllor2 $\wedge$ directionup $\wedge$ ButtonPressed5 $\rightarrow$ (directionup U floor5)
- It is impossible to get to a state where a system has started but is not ready
  - ϕ = G $\neg$(started $\wedge$ $\neg$ready)
  - What is the meaning of (intuitive) negation of ϕ ?
    - For every path, it is possible to get to such a state (started$\wedge\neg$ready).
    - There exists a such path that gets to such a state.
      - we cannot express this meaning directly
- LTL has limited expressive power
  - For example, LTL cannot express statements which assert the existence of a path
    - From any state s, there exists a path π starting from s to get to a restart state
    - The lift can remain idle on the third floor with its doors closed
  - Computation Tree Logic (CTL) has operators for quantifying over paths and can express these properties

Summary of practical patterns

G p	always p	invariance
F p	eventually p	guarantee
$p \rightarrow (F\ q)$	p implies eventually q	response
$p \rightarrow (q\ U\ r)$	p implies q until r	precedence
G F p	always, eventually p	recurrence (progress)
F G p	eventually, always p	stability (non-progress)
$F\ p \rightarrow F\ q$	eventually p implies eventually q	correlation

Equivalences between LTL formulas

- Def 3.9 $\phi \equiv \psi$ if for all models $\mathcal{M}$ and all paths π in $\mathcal{M}$: $\pi \vDash \phi$ iff $\pi \vDash \psi$
- $\neg G\ \phi \equiv F\ \neg\phi$, $\neg F\ \phi \equiv G\ \neg\phi$, $\neg X\ \phi \equiv X\ \neg\phi$
- $\neg\ (\phi\ U\ \psi) \equiv \neg\phi\ R\ \neg\psi$, $\neg(\phi\ R\ \psi) \equiv \neg\phi\ U\ \neg\psi$
- $F\ (\phi \lor \psi) \equiv F\ \phi \lor F\ \psi$
- $G\ (\phi \land \psi) \equiv G\ \phi \land G\ \psi$
- $F\ \phi \equiv T\ U\ \phi$, $G\ \phi \equiv \bot\ R\ \phi$
- $\phi\ U\ \psi \equiv \phi\ W\ \psi \land F\ \psi$
- $\phi\ W\ \psi \equiv \phi\ U\ \psi \lor G\ \phi$
- $\phi\ W\ \psi \equiv \psi\ R\ (\phi \lor \psi)$
- $\phi\ R\ \psi \equiv \psi\ W\ (\phi \land \psi)$

Adequate sets of connectives for LTL (1/2)

- X is completely orthogonal to the other connectives
  - X does not help in defining any of the other connectives.
  - The other way is neither possible
- Each of the sets {U,X}, {R,x}, {W,X} is adequate
  - {U,X}
    - ϕ R $\psi \equiv \neg$ ($\neg$ ϕ U $\neg$ ψ)
    - ϕ W $\psi \equiv \psi$ R ($\phi \vee \psi$) $\equiv \neg$ ($\neg\psi$ U $\neg$($\phi \vee \psi$))
  - {R,X}
    - ϕ U $\psi \equiv \neg$ ($\neg\phi$ R $\neg\psi$)
    - ϕ W $\psi \equiv \psi$ R ($\phi \vee \psi$)
  - {W,X}
    - ϕ U $\psi \equiv \neg$ ($\neg$ ϕ R $\neg$ ψ)
    - ϕ R $\psi \equiv \psi$ W ($\phi \wedge \psi$)

Adequate sets of connectives for LTL (2/2)

- Thm 4.10 ϕ U $\psi \equiv \neg(\neg\psi$ U $(\neg\phi \wedge \neg\psi)) \wedge$ F ψ
- Proof: take any path $s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \ldots$ in any model
  - Suppose $s_0 \vDash \phi$ U ψ
    - Let n be the smallest number s.t. $s_n \vDash \psi$
      - We know that such n exists from ϕ U ψ. Thus, $s_0 \vDash$ F ψ
      - For each k < n, $s_k \vDash \phi$ since ϕ U ψ
    - We need to show $s_0 \vDash \neg(\neg\psi$ U $(\neg\phi \wedge \neg\psi))$
      - case 1: for all i, $s_i \nvDash \neg\phi \wedge \neg\psi$. Then, $s_0 \vDash \neg(\neg\psi$ U $(\neg\phi \wedge \neg\psi))$
      - case 2: for some i, $s_i \vDash \neg\phi \wedge \neg\psi$. Then, we need to show
        - (*)for each i >0, if $s_i \vDash \neg\phi \wedge \neg\psi$, then there is some j < i with $s_j \nvDash \neg\psi$ (i.e. $s_j \vDash \psi$)
        - Take any i >0 with $s_i \vDash \neg\phi \wedge \neg\psi$. We know that i > n since $s_0 \vDash \phi$ U ψ. So we can take j=n and have $s_j \vDash \psi$
  - Conversely, suppose $s_0 \vDash \neg(\neg\psi$ U $(\neg\phi \wedge \neg\psi)) \wedge$ F ψ
    - Since $s_0 \vDash$ F ψ, we have a minimal n as before s.t. $s_n \vDash \psi$
      - case 1: for all i, $s_i \nvDash \neg\phi \wedge \neg\psi$ (i.e. $s_i \vDash \phi \vee \psi$). Then $s_0 \vDash \phi$ U ψ
      - case 2: for some i, $s_i \vDash \neg\phi \wedge \neg\psi$. We need to prove for any i <n, $s_i \vDash \phi$
        - Suppose $s_i \nvDash \phi$ (i.e., $s_i \vDash \neg\phi$). Since n is minimal, we know $s_i \vDash \neg\psi$. So by (*) there is some j <i<n with $s_j \vDash \psi$, contradicting the minimality of n. Contradiction