Software Model Checking I

Dynamic v.s. Static Analysis

	Dynamic Analysis (i.e., testing)	Static Analysis (i.e. model checking)
Pros	•Real result •No environmental limitation •Binary library is ok	•Complete analysis result •Fully automatic •Concrete counter example
Cons	•Incomplete analysis result •Test case selection	•Consumed huge memory space •Takes huge time for verification •False alarms

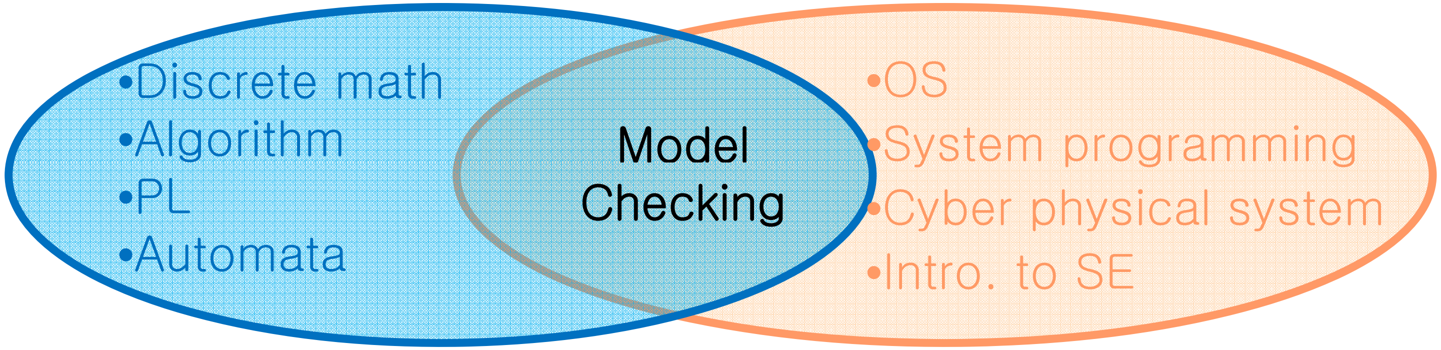
Motivation for Software Model Checking

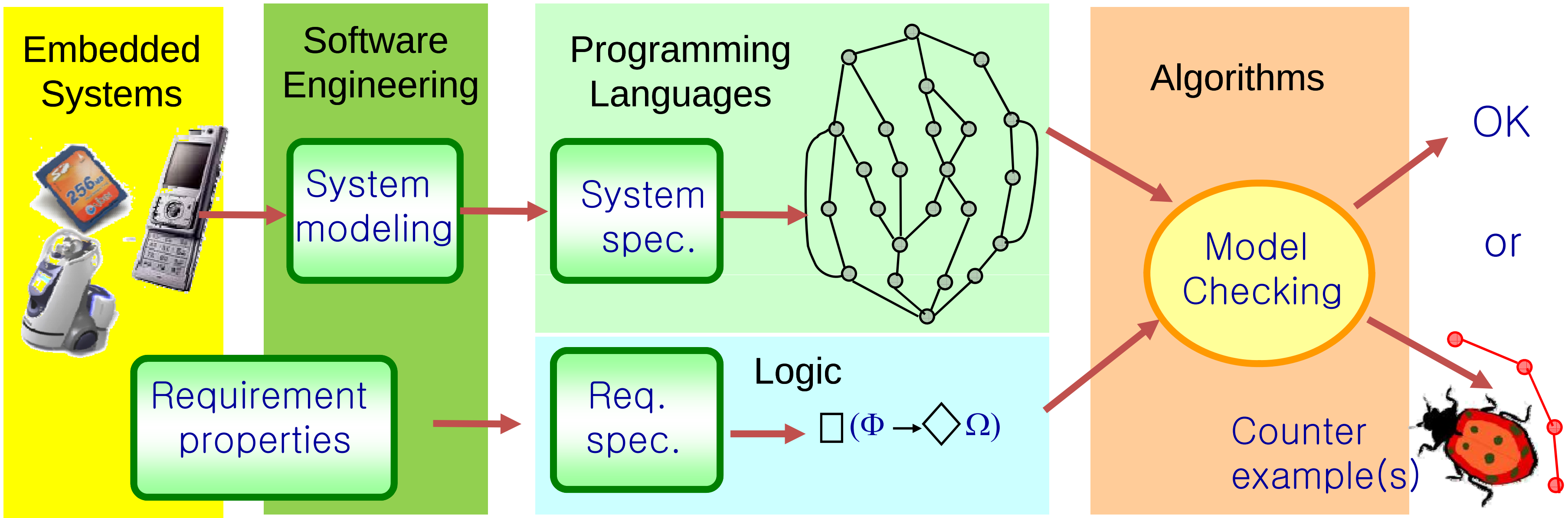
- Data flow analysis (DFA): fastest & least precision
  - "May" analysis,
- Abstract interpretation (AI): fast & medium precision
  - Over-approximation & under-approximation
- Model checking (MC): slow & complete
  - Complete value analysis
  - No approximation

- Static analyzer & MC as a C debugger
  - Handling complex C structures such as pointer and array
    - DFA: might-be
    - AI: may-be
    - MC: can-be or should-be

Model Checking Background

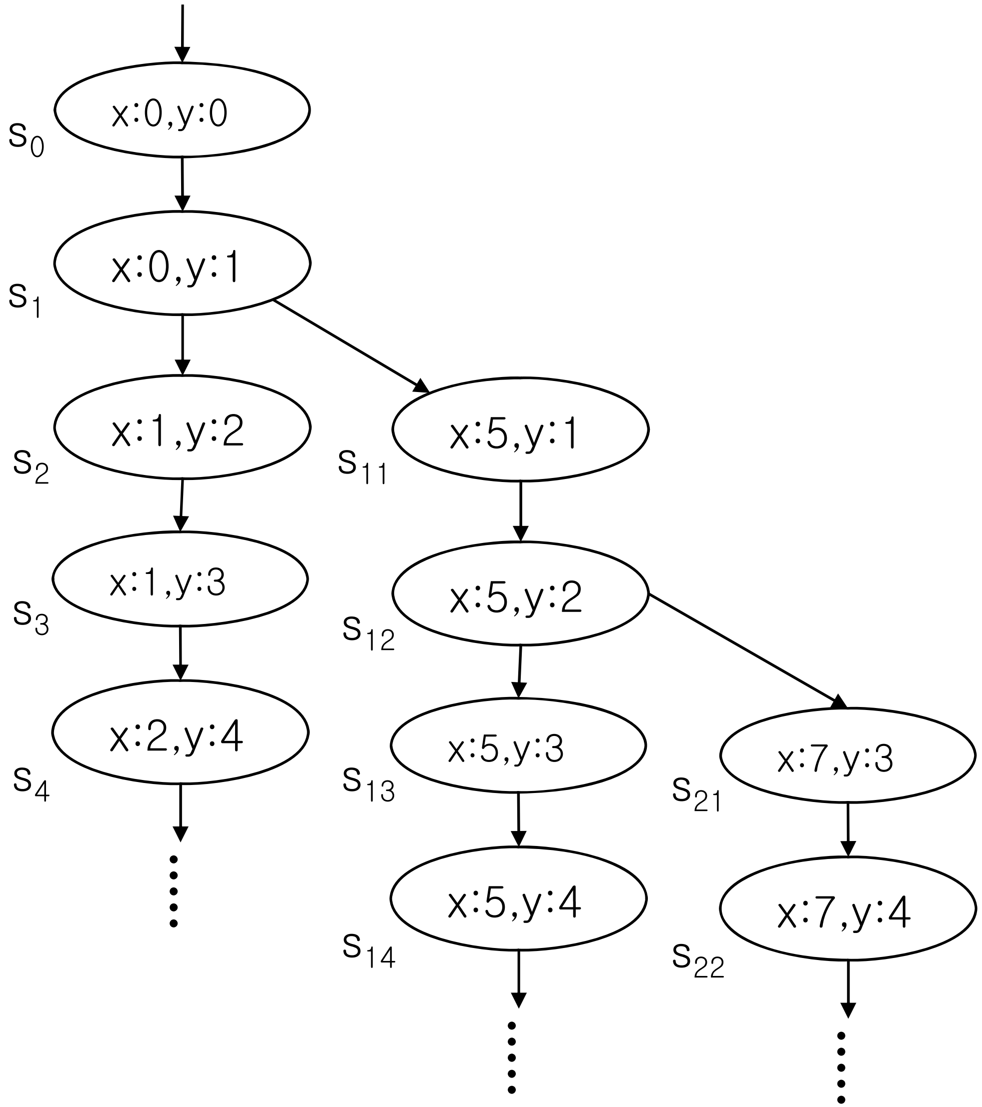
- Undergraduate CS classes contributing to this area





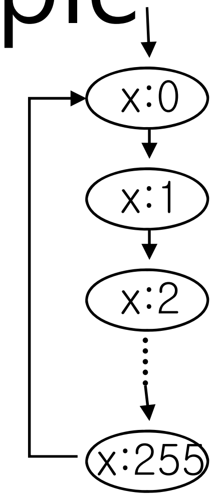
Operational Semantics of Software

- A system execution σ is a sequence of states $s_0 s_1 \ldots$
  - A state has an environment ρ_s: *Var-> Val*
- A system has its semantics as a set of system executions



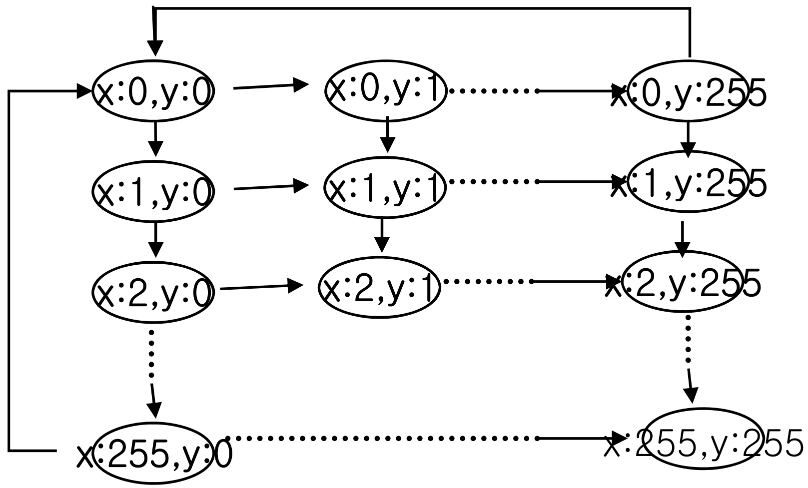
Example

```
active type A() {
byte x;
again:
    x++;
    goto again;
}
```




```
active type A() {
byte x;
again:
    x++;
    goto again;
}

active type B() {
byte y;
again:
    y++;
   goto again;
}
```



Pros and Cons of Model Checking

- Pros
  - Fully automated and provide complete coverage
  - Concrete counter examples
  - Full control over every detail of system behavior
    - Highly effective for analyzing
      - embedded software
      - multi-threaded systems
- Cons
  - State explosion problem
  - An abstracted model may not fully reflect a real system
  - Needs to use a specialized modeling language
    - Modeling languages are similar to programming languages, but simpler and clearer

Companies Working on Model Checking

Microsoft

SAMSUNG

INRIA

IBM

ESTEREL Technologies

cadence

NEC Empowered by Innovation

Sun microsystems

Jet Propulsion Laboratory
California Institute of Technology

The MathWorks
Accelerating the pace of engineering and science

Model Checking History

1981	Clarke / Emerson: CTL Model Checking Sifakis / Quielle	10^5
1982	EMC: Explicit Model Checker Clarke, Emerson, Sistla	
1990	Symbolic Model Checking Burch, Clarke, Dill, McMillan	10^{100}
1992	SMV: Symbolic Model Verifier McMillan	
1998	Bounded Model Checking using SAT Biere, Clarke, Zhu	10^{1000}
2000	Counterexample-guided Abstraction Refinement Clarke, Grumberg, Jha, Lu, Veith	

Example. Sort (1/2)

9	14	2	255

- Suppose that we have an array of 4 elements each of which is 1 byte long
  - unsigned char a[4];
- We wants to verify sort.c works correctly
  - main() { sort(); assert(a[0]<= a[1]<= a[2]<=a[3]);}
- Hash table based explicit model checker (ex. Spin) generates at least 2^{32} (= $4x10^{9}$ = 4G) states
    - 4G states x 4 bytes = 16 Gbytes, no way…
- Binary Decision Diagram (BDD) based symbolic model checker (ex. NuSMV) takes 200 MB in 400 sec

Example. Sort (2/2)

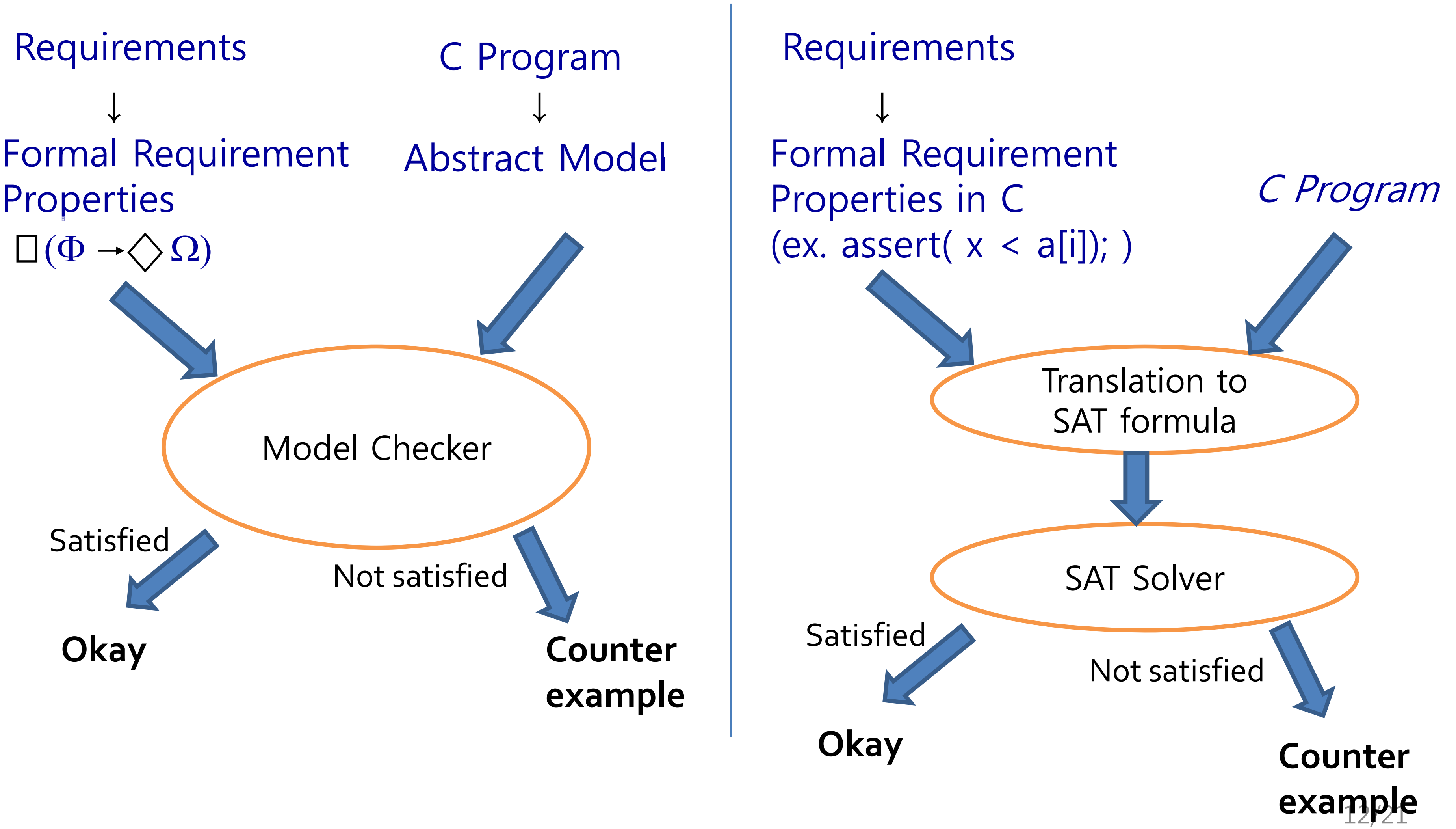
```
#include <stdio.h>
#define N 5
int main(){
    int data[N], i, j, tmp;
    /* Assign random values to the array*/
    for (i=0; i<N; i++){
        data[i] = nondet_int();
    }
   /* It misses the last element, i.e., data[N-1]*/
    for (i=0; i<N-1; i++)
        for (j=i+1; j<N-1; j++)
            if (data[i] > data[j]){
                tmp = data[i];
                data[i] = data[j];
                data[j] = tmp;
            }
/* Check the array is sorted */
    for (i=0; i<N-1; i++){
        assert(data[i] <= data[i+1]);
    }
}
```

- SAT-based Bounded Model Checker
  - Total 19099 CNF clause with 6224 boolean propositional variables
  - Theoretically, 2^{6224} choices should be evaluated!!!

SAT	VSIDS
Conflicts	73
Decisions	2435
Time(sec)	0.015

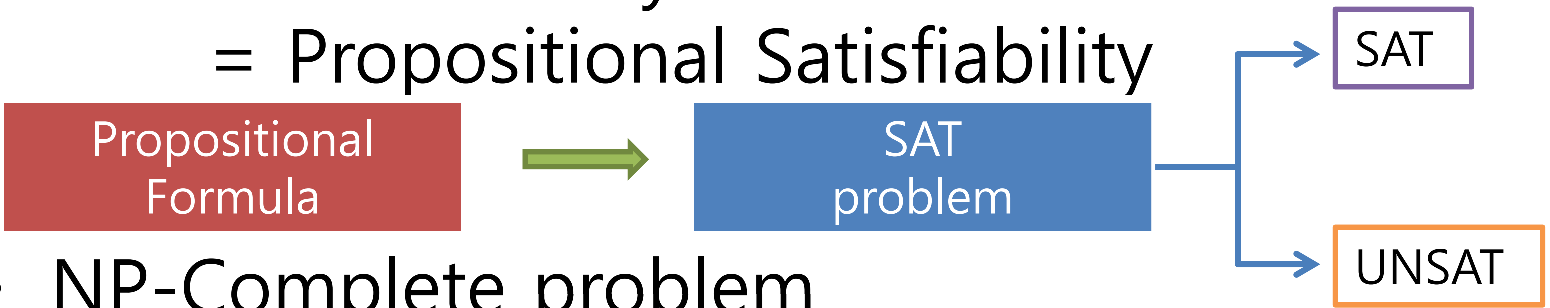
UNSAT	VSIDS
Conflicts	35067
Decisions	161406
Time(sec)	1.89

Overview of SAT-based Bounded Model Checking



SAT Basics (1/3)

- SAT = Satisfiability
  = Propositional Satisfiability



- NP-Complete problem
  - We can use SAT solver for many NP-complete problems
    - Hamiltonian path
    - 3 coloring problem
    - Traveling sales man's problem
- Recent interest as a verification engine

SAT Basics (2/3)

- A set of propositional variables and Conjunctive Normal Form (CNF) clauses involving variables
  - $(x_1 \vee x_2' \vee x_3) \wedge (x_2 \vee x_1' \vee x_4)$
  - x_1, x_2, x_3 and x_4 are variables (true or false)
- Literals: Variable and its negation
  - x_1 and x_1'
- A clause is satisfied if one of the literals is true
  - x_1=true satisfies clause 1
  - x_1=false satisfies clause 2
- Solution: An assignment that satisfies all clauses

SAT Basics (3/3)

- DIMACS SAT Format
  - Ex. $(x_1 \vee x_2' \vee x_3)$

    $\wedge (x_2 \vee x_1' \vee x_4)$

```
p cnf 4 2
1 -2 3 0
2 -1 4 0
```

Model

x_1	x_2	x_3	x_4	Formula
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	T
T	F	T	T	T
T	F	T	F	F
T	F	F	T	T
T	F	F	F	F
F	T	T	T	T
F	T	T	F	T
F	T	F	T	F
F	T	F	F	F
F	F	T	T	T
F	F	T	F	T
F	F	F	T	T
F	F	F	F	T

Software Model Checking as a SAT problem (1/3)

- Unwinding Loop

Original code

```
x=0;
while (x < 2) {
  y=y+x;
  x++;
}
```

Unwinding the loop 3 times

```
x=0;
if (x < 2) {
  y=y+x;
  x++;
}
if (x < 2) {
  y=y+x;
  x++;
}
if (x < 2) {
  y=y+x;
  x++;
}
```

Unwinding assertion: →

```
assert (! (x < 2))
```

Examples

```
/* Straight-forward
   constant upperbound */
for(i=0,j=0; i < 5; i++) {
    j=j+i;
}
```

```
/*Constant upperbound*/
for(i=0,j=0; j < 10; i++) {
    j=j+i;
}
```

```
/* Complex upperbound */
for(i=0; i < 5; i++) {
    for(j=i; j < 5;j++) {
        for(k= i+j; k < 5; k++) {
            m += i+j+k;
        }
    }
}
```

```
/* Upperbound unknown */
for(i=0,j=0; i^6-4*i^5 -17*i^4 != 9604 ; i++) {
    j=j+i;
}
```

Model Checking as a SAT problem (2/3)

- From C Code to SAT Formula

Original code

```
x=x+y;
if (x!=1)
  x=2;
else
  x++;
assert(x<=3);
```

Convert to static single assignment (SSA)

$$
\begin{aligned}
&x_1 = x_0 + y_0; \\
&\texttt{if } (x_1 != 1) \\
&\quad x_2 = 2; \\
&\texttt{else} \\
&\quad x_3 = x_1 + 1; \\
&x_4 = (x_1 != 1) ? x_2 : x_3; \\
&\texttt{assert}(x_4 <= 3);
\end{aligned}
$$

Generate constraints

$$C \equiv x_1 = x_0 + y_0 \land x_2 = 2 \land x_3 = x_1 + 1 \land (x_1 != 1 \land x_4 = x_2 \lor x_1 = 1 \land x_4 = x_3)$$

$$P \equiv x_4 <= 3$$

Check if $C \land \neg P$ is satisfiable, if it is then the assertion is violated

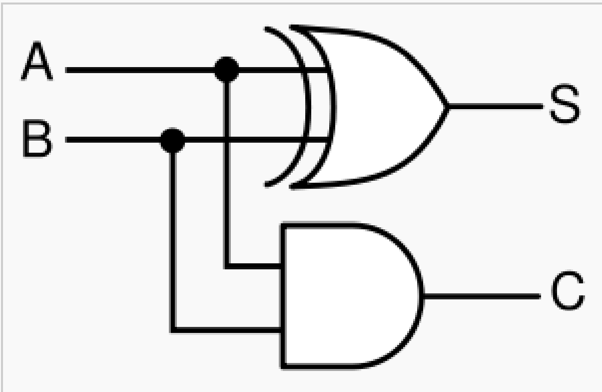
$C \land \neg P$ is converted to Boolean logic using a bit vector representation for the integer variables $y_0, x_0, x_1, x_2, x_3, x_4$

Model Checking as a SAT problem (3/3)

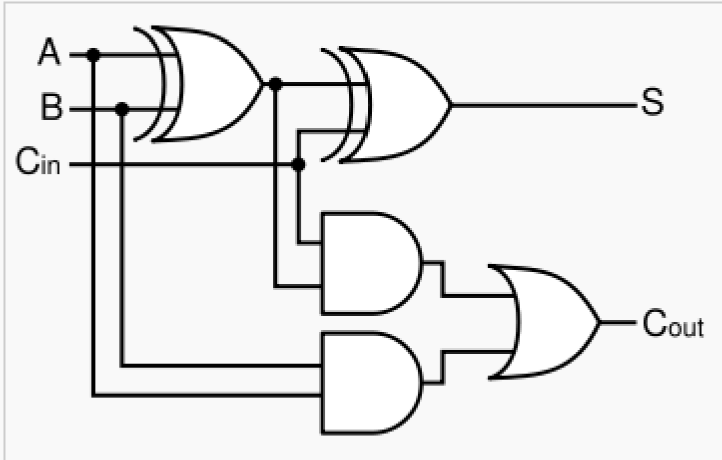
- Example of arithmetic encoding into pure propositional formula

  Assume that x,y,z are three bits positive integers represented by propositions $x_0x_1x_2$, $y_0y_1y_2$, $z_0z_1z_2$

  $$C \equiv z=x+y \equiv \ (z_0 \leftrightarrow (x_0 \oplus y_0) \oplus ((x_1 \wedge y_1) \vee (((x_1 \oplus y_1) \wedge (x_2 \wedge y_2)))$$
  $$\wedge\ (z_1 \leftrightarrow (x_1 \oplus y_1) \oplus (x_2 \wedge y_2))$$
  $$\wedge\ (z_2 \leftrightarrow (x_2 \oplus y_2))$$



Half adder circuit diagram



Full adder circuit diagram
Inputs: {A, B, CarryIn} → Outputs: {Sum, CarryOut}

Example

```
/* Assume that x and y are 2 bit
unsigned integers */
/* Also assume that x+y <= 3 */
void f(unsigned int y) {
    unsigned int x=1;
    x=x+y;
    if (x==2)
        x+=1;
    else
        x=2;
    assert(x ==2);
}
```

Model Checking as a SAT problem (4/4)

- CBMC (C Bounded Model Checker)
  - Handles function calls using inlining
  - A return value of a undefined function is considered non-deterministic
  - Unwinds the loops a fixed number of times
  - Models various scenarios using non-determinism
    - So that a program can be checked for a set of inputs rather than a single input
    - `__CPROVER_assume( x> 0);`
  - Allows specification of assertions which are checked using the bounded model checking