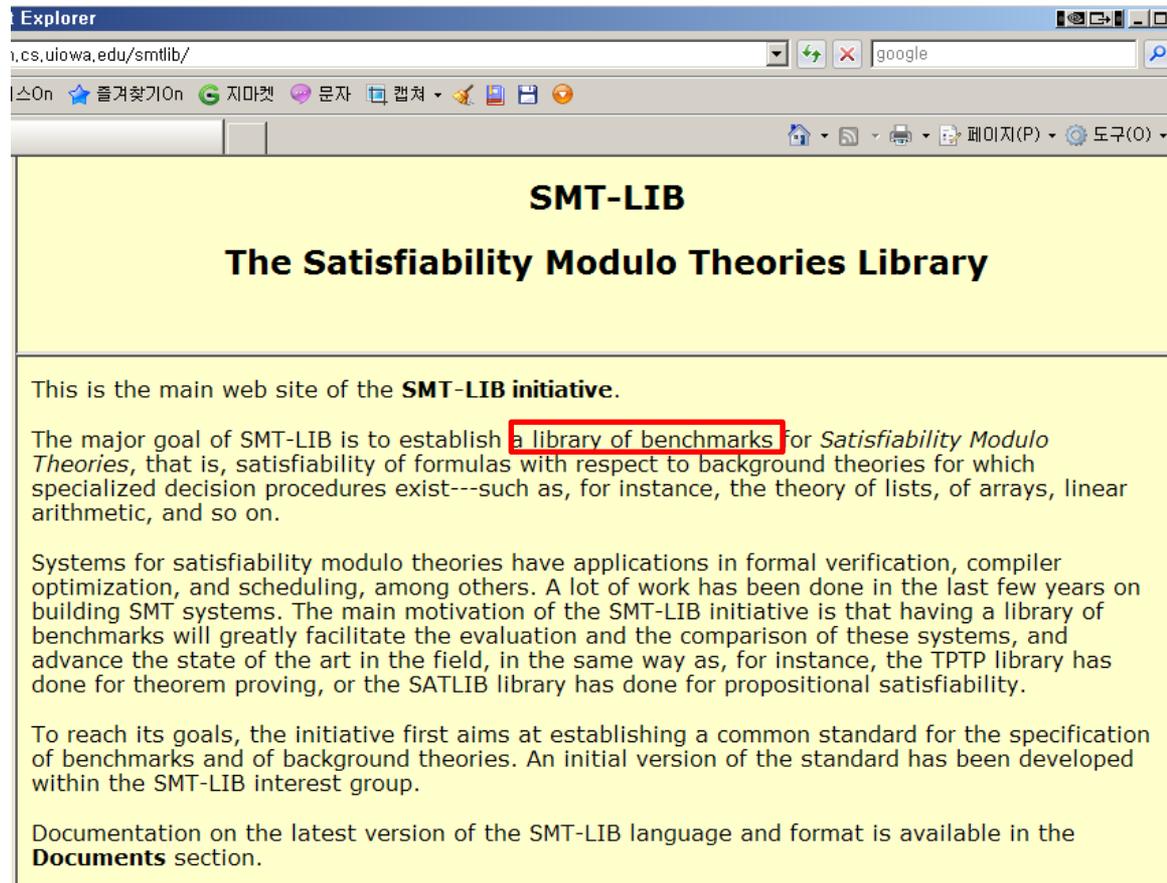


The Satisfiability Modulo Theories Library (SMT-LIB)

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The screenshot shows a web browser window with the address bar containing "i.cs.uiowa.edu/smtlib/". The browser's toolbar includes icons for home, back, forward, and search, along with a search engine dropdown set to "google". The website content is displayed on a yellow background. At the top, it reads "SMT-LIB" in bold black text, followed by "The Satisfiability Modulo Theories Library" in a slightly smaller bold black font. Below this, a paragraph states: "This is the main web site of the **SMT-LIB initiative**." The next paragraph explains the goal: "The major goal of SMT-LIB is to establish **a library of benchmarks** for Satisfiability Modulo Theories, that is, satisfiability of formulas with respect to background theories for which specialized decision procedures exist---such as, for instance, the theory of lists, of arrays, linear arithmetic, and so on." A third paragraph describes the applications: "Systems for satisfiability modulo theories have applications in formal verification, compiler optimization, and scheduling, among others. A lot of work has been done in the last few years on building SMT systems. The main motivation of the SMT-LIB initiative is that having a library of benchmarks will greatly facilitate the evaluation and the comparison of these systems, and advance the state of the art in the field, in the same way as, for instance, the TPTP library has done for theorem proving, or the SATLIB library has done for propositional satisfiability." A fourth paragraph discusses the standardization process: "To reach its goals, the initiative first aims at establishing a common standard for the specification of benchmarks and of background theories. An initial version of the standard has been developed within the SMT-LIB interest group." The final paragraph mentions documentation: "Documentation on the latest version of the SMT-LIB language and format is available in the **Documents** section."

Supported Theories (SMT-Lib v2)

- [ArraysEx](#)
 - Functional arrays with extensionality.
- [Fixed Size BitVectors](#)
 - Bit vectors with arbitrary size.
- **Core**
 - Core theory, defining the basic Boolean operators
- [Ints](#)
 - Integer numbers.
- [Reals](#)
 - Real numbers.
- [Reals Ints](#)
 - Real and integer numbers.

Supported Sublogics

AUFLIA: Closed formulas over the theory of linear integer arithmetic and arrays extended with free sort and function symbols but restricted to arrays with integer indices and values.

AUFLIRA: Closed linear formulas with free sort and function symbols over one- and two-dimensional arrays of integer index and real value.

AUFNIRA: Closed formulas with free function and predicate symbols over a theory of arrays of arrays of integer index and real value.

LRA: Closed linear formulas in linear real arithmetic.

QF ABV: Closed quantifier-free formulas over the theory of bitvectors and bitvector arrays.

QF AUFBV: Closed quantifier-free formulas over the theory of bitvectors and bitvector arrays extended with free sort and function symbols.

QF AUFLIA: Closed quantifier-free linear formulas over the theory of integer arrays extended with free sort and function symbols.

QF AX: Closed quantifier-free formulas over the theory of arrays with extensionality.

QF BV: Closed quantifier-free formulas over the theory of fixed-size bitvectors.

QF IDL: Difference Logic over the integers. In essence, Boolean combinations of inequations of the form $x - y < b$ where x and y are integer variables and b is an integer constant.

QF LIA: Unquantified linear integer arithmetic. In essence, Boolean combinations of inequations between linear polynomials over integer variables.

QF LRA: Unquantified linear real arithmetic. In essence, Boolean combinations of inequations between linear polynomials over real variables.

QF NIA: Quantifier-free integer arithmetic.

QF NRA: Quantifier-free real arithmetic.

QF RDL: Difference Logic over the reals. In essence, Boolean combinations of inequations of the form $x - y < b$ where x and y are real variables and b is a rational constant.

QF UF: Unquantified formulas built over a signature of uninterpreted (i.e., free) sort and function symbols.

QF UFBV: Unquantified formulas over bitvectors with uninterpreted sort function and symbols.

QF UFIDL: Difference Logic over the integers (in essence) but with uninterpreted sort and function symbols.

QF UFLIA: Unquantified linear integer arithmetic with uninterpreted sort and function symbols.

QF UFLRA: Unquantified linear real arithmetic with uninterpreted sort and function symbols.

QF UFNRA: Unquantified non-linear real arithmetic with uninterpreted sort and function symbols.

UFLRA: Non-linear real arithmetic with uninterpreted sort and function symbols.

UFNIA: Non-linear integer arithmetic with uninterpreted sort and function symbols.

Theory of Arrays

(theory Arrays

:written_by {Silvio Ranise and Cesare Tinelli}

:date {08/04/05}

:sorts (Index Element Array)

Predefined
data types

:funs ((select Array Index Element)

(store Array Index Element Array))

Predefined f
unctions

:definition

"This is a theory of functional arrays without extensionality.
It is formally and completely defined by the axioms below."

:axioms (

(forall (?a Array) (?i Index) (?e Element)

(= (select (store ?a ?i ?e) ?i) ?e))

Bounded
variables

(forall (?a Array) (?i Index) (?j Index) (?e Element)

(or (= ?i ?j)

(= (select (store ?a ?i ?e) ?j) (select ?a ?j))))

Prefix
operator

:notes

"It is not difficult to prove that the two
axioms above are logically equivalent
to the following \"McCarthy axiom\":

(forall (?a Array) (?i Index) (?j Index)

(?e Element)

(= (select (store ?a ?i ?e) ?j)

(ite (= ?i ?j) ?e (select ?a ?j))))

If-then-else
term construct

Such an axiom appeared in the following
paper:

Correctness of a Compiler for Arithmetic
Expressions,

by John McCarthy and James Painter,
available at <http://www-formal.stanford.edu/jmc/mcpain.html>.

"

)

Theory of Arrays w/ Extensionability

```
(theory ArraysEx
:written_by {Silvio Ranise and Cesare Tinelli}
:date {08/04/05}
:updated {28/10/05}
:history {
Bug fix in the third axiom, pointed out by Robert
Nieuwenhuis:
The scope of 'forall (?i Index)' was the whole implication
instead of just the premise of the implication.
}
:sorts (Index Element Array)
:funs ((select Array Index Element)
(store Array Index Element Array))
:definition
"This is a theory of functional arrays with extensionality.
It is formally and completely defined by the axioms below.
"
```

```
:axioms
(
(forall (?a Array) (?i Index) (?e Element)
(= (select (store ?a ?i ?e) ?i) ?e))
(forall (?a Array) (?i Index) (?j Index) (?e Element)
(or (= ?i ?j)
(= (select (store ?a ?i ?e) ?j)
(select ?a ?j))))
(forall (?a Array) (?b Array)
(implies (forall (?i Index) (= (select ?a ?i) (select ?b ?i)))
(= ?a ?b)))
)
:notes "This theory extends the theory Arrays with
an axiom stating that any two arrays with the same
elements are in fact the same array. "
```

Theory of Integer

(theory Ints

:sorts (Int)

:notes

"The (unsupported) annotations of the function/predicate symbols have

the following meaning:

attribute | possible value | meaning

:assoc // the symbol is associative
:comm // the symbol is commutative
:unit a constant
:trans // the symbol is transitive
:refl // the symbol is reflexive
:irref // the symbol is irreflexive
:antisym // the symbol is antisymmetric

"

:funs ((0 Int)

(1 Int)

(~ Int Int) ; unary minus

(- Int Int Int) ; binary minus

(+ Int Int Int :assoc :comm :unit {0})

(* Int Int Int :assoc :comm :unit {1}))

:preds ((<= Int Int :refl :trans :antisym)

(< Int Int :trans :irref)

(>= Int Int :refl :trans :antisym)

(> Int Int :trans :irref)

)

:definition

"This is the first-order theory of the integers, that is , the set of all the first-order sentences over the given signature that are true in the structure of the integer numbers interpreting the signature's symbols in the obvious way (with ~ denoting the negation and - the subtraction functions). "

:notes

"Note that this theory is **not** (recursively) axiomatizable in a first-order logic such as SMT-LIB's underlying logic. That is why the theory is defined semantically. "

)

Example of QF_LIA Benchmark

(benchmark example

:status sat

Expected output (optional)

:logic QF_LIA

Theory

:extrafuns ((x1 Int)

User defined variables

(x2 Int) (x3 Int) (x4 Int) (x5 Int))

;human readable form

Comments

; x1-1 >= x2 /\

; x1-3 <= x2 /\

; x1 = 2 x3+x5 /\

; x3 = x5 /\

; x2 = 6 x4

:formula (and

Target formula

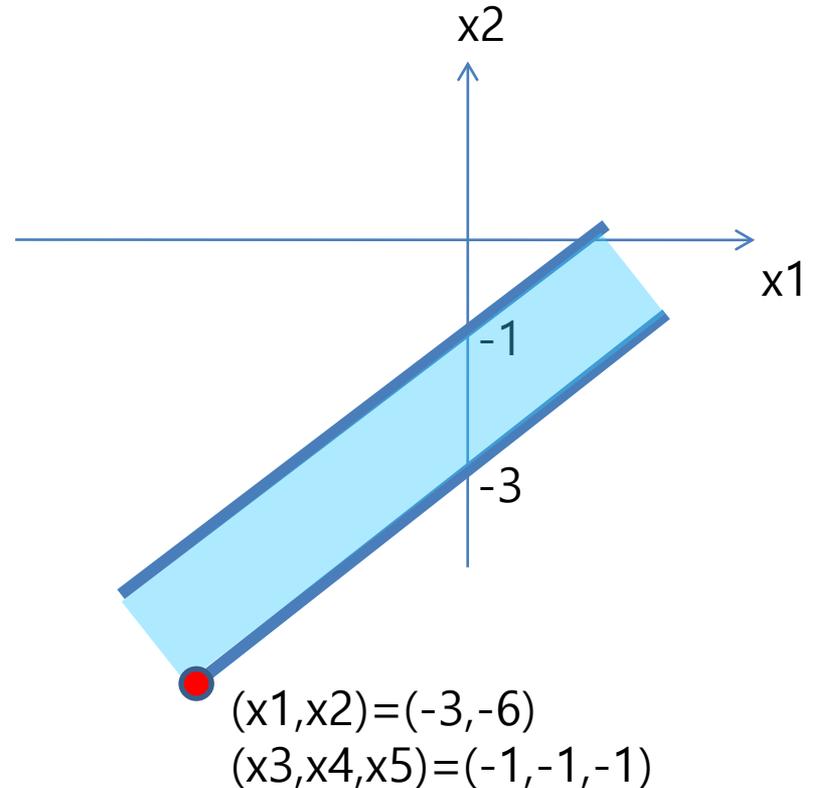
(>= (- x1 x2) 1)

(<= (- x1 x2) 3)

(= x1 (+ (* 2 x3) x5))

(= x3 x5)

(= x2 (* 6 x4))))



Example of QF_UF Benchmark

(benchmark example2-1

:logic QF_UF

:**extrasorts** (A B C D)

User defined
data types

:extrafuns ((x A)(y B)(w A)(z C)(u D))

:extrafuns ((**f** A A B)

User defined functions

(g A B B) (h1 B A B) (h2 B B B))

;human readable form

; g(x,y) = h1(y,x) \wedge

; f(x,x) = h2(y,y) \wedge

; f(x,x) != f(x,w)

:assumption((= (g x y) (h1 y x)))

:assumption((= (f x x) (h2 y y)))

:assumption((not (= (f x x) (f x w))))

:formula true

)

A model for the formula

x-> v0

y->v1

w->v4

g->{ (v1,v0)->v2,
 else-> v2}

f->{(v0,v0)->v3,
 (v0,v4)->v5,
 else->v5}

h2->{(v1,v1)->v3,
 else -> v3}

Another Example of QF_UF Benchmark

1. Prove that

$F : a=b \wedge b=c \rightarrow g(f(a), b) = g(f(c), a)$

is T_{EUF} -**satisfiable**

2. Prove

$F : a=b \wedge b=c \rightarrow g(f(a), b) = g(f(c), a)$

is T_{EUF} -**valid** through proof tree

(benchmark Quiz2

:logic QF_UF

:extrasorts (A B C)

:extrafuns ((a A) (b A) (c A))

:extrafuns ((f A B) (g B A C))

;human readable form

; a=b/\ b=c -> g(f(a),b) = g(f(c),a)

:formula (implies (and (= a b) (= b c))

(= (g (f a) b) (g (f c) a))))

**Then, how to check
 T_{EUF} -validity of F by
using a SMT solver???**

A model for the formula

a->v0

b->v1

c->v2

f->{v0->v3,v2->v5,else->v5}

g->{(v3,v1)->v4,(v5,v0)->v6,else->v6}

Example of QF_AUFLIA

(benchmark sort

:logic QF_AUFLIA

:extrafuns ((data_7 Array)) ; **initial data[] declaration**

:extrafuns ((tmp_0 Int))

:extrafuns ((i_9 Int))

:assumption (= i_9 0) ; **i=0;**

:extrafuns ((j_1 Int))

:assumption (= j_1 1) ; **j=1;**

:extrafuns ((tmp_1 Int))

:assumption (= tmp_1 (select data_7 0)) ; **tmp = data[0]**

:extrafuns ((data_8 Array))

:assumption (= data_8 (store data_7 0 (select data_7 1))) ; **data[0]=data[1];**

:extrafuns ((data_9 Array))

:assumption (= data_9 (store data_8 1 tmp_1)) ; **data[1] = tmp;**

:extrafuns ((data_10 Array))

; if (data[0] > data[1]) { tmp=data[0]; data[0]=data[1]; data[1]=tmp }

:assumption (= data_10 (if_then_else (> (select data_7 0) (select data_7 1)
) data_9 data_7))

...

:formula (**not**

(and (<= (select data_70 0) (select data_70 1))

(<= (select data_70 1) (select data_70 2))

...)

10/14

```
#define N 7
int main(){
  int data[N], i, j, tmp;
  for (i=0; i<N-1; i++)
    for (j=i+1; j<N; j++)
      if(data[i]>data[j]){
        tmp = data[i];
        data[i] = data[j];
        data[j] = tmp;
      }
  assert(data[0]<=data[1]&&...);
}
```

Theory of Fixed_Size_BitVectors[32]

:sorts_description

"All sort symbols of the form $\text{BitVec}[i]$,
where i is a numeral between 1 and 32, inclusive."

:funs_description

"All function symbols with arity of the form
 $(\text{concat } \text{BitVec}[i] \text{BitVec}[j] \text{BitVec}[m])$ where
- i, j, m are numerals
- $i, j > 0$
- $i + j = m \leq 32$ "

:funs_description

"All function symbols with arity of the form
 $(\text{extract}[i:j] \text{BitVec}[m] \text{BitVec}[n])$ where
- i, j, m, n are numerals
- $32 \geq m > i \geq j \geq 0$,
- $n = i - j + 1$. "

:funs_description

"All function symbols with arity of the form
 $(\text{op1 } \text{BitVec}[m] \text{BitVec}[m])$ or
 $(\text{op2 } \text{BitVec}[m] \text{BitVec}[m] \text{BitVec}[m])$ where
- op1 is from $\{\text{bvnot}, \text{bvneg}\}$
- op2 is from $\{\text{bvand}, \text{bvor}, \text{bvxor}, \text{bvsub}, \text{bvadd}, \text{bvmul}\}$
- m is a numeral
- $0 < m \leq 32$ "

:preds_description

"All predicate symbols with arity of the form
 $(\text{pred } \text{BitVec}[m] \text{BitVec}[m])$ where
- pred is from $\{\text{bvlt}, \text{bvleq}, \text{bvgeq}, \text{bvgt}\}$
- m is a numeral
- $0 < m \leq 32$ "

- Variables

If v is a variable of sort $\text{BitVec}[m]$ with $0 < m \leq 32$, then
 $[[v]]$ is some element of $\{0, \dots, m-1\} \rightarrow \{0, 1\}$, the set of
total functions from $\{0, \dots, m-1\}$ to $\{0, 1\}$.

- Constant symbols bv0 and bv1 of sort $\text{BitVec}[32]$

$[[\text{bv0}]] := \lambda x : [0..32]. 0$

$[[\text{bv1}]] := \lambda x : [0..32]. \text{if } x = 0 \text{ then } 1 \text{ else } 0$

- Function symbols for concatenation

$[[\text{concat } s \ t]] := \lambda x : [0..n+m].$

$\text{if } (x < m) \text{ then } [[t]](x) \text{ else } [[s]](x-m)$ where

s and t are terms of sort $\text{BitVec}[n]$ and $\text{BitVec}[m]$,
respectively, $0 < n \leq 32$, $0 < m \leq 32$, and $n+m \leq 32$.

- Function symbols for extraction

$[[\text{extract}[i:j] \ s]] := \lambda x : [0..i-j+1]. [[s]](j+x)$

where s is of sort $\text{BitVec}[l]$, $0 \leq j \leq i < l \leq 32$.

- Function symbols for arithmetic operations

To define the semantics of the bitvector operators bvadd ,
 bvsub , bvneg , and bvmul , it is helpful to use these
ancillary functions:

o bv2nat which takes a bitvector $b : [0..m] \rightarrow \{0, 1\}$

with $0 < m \leq 32$, and returns an integer in the range
 $[0..2^m)$, and is defined as follows:

$\text{bv2nat}(b) := b(m-1) \cdot 2^{m-1} + b(m-2) \cdot 2^{m-2} + \dots + b(0) \cdot 2^0$

o $\text{nat2bv}[m]$, with $0 < m \leq 32$, which takes a non-negative

integer n and returns the (unique) bitvector $b : [0..m] \rightarrow \{0, 1\}$

such that $b(m-1) \cdot 2^{m-1} + \dots + b(0) \cdot 2^0 = n \text{ MOD } 2^m$

where MOD is usual modulo operation.

$[[\text{bvadd } s \ t]] := \text{nat2bv}[m](\text{bv2nat}(s) + \text{bv2nat}(t))$

SMTLIB Benchmark Syntax

- Reserved keywords

- =, and, benchmark, distinct, exists, false, flet, forall, if then else, iff, implies, ite, let, logic, not, or, sat, theory, true, unknown, unsat, xor

Formulas

```
<prop_atom> ::= true | false | <fvar> | <identifier>
<an_atom> ::= <prop_atom> | ( <prop_atom> <annotation>+ )
           | ( <pred_symb> <an_term>+ <annotation>* )

<connective> ::= not | implies | if_then_else
              | and | or | xor | iff

<quant_symb> ::= exists | forall
<quant_var> ::= ( <var> <sort_symb> )
<an_formula> ::= <an_atom>
              | ( <connective> <an_formula>+ <annotation>* )
              | ( <quant_symb> <quant_var>+ <an_formula> <annotation>* )
              | ( let ( <var> <an_term> ) <an_formula> <annotation>* )
              | ( flet ( <fvar> <an_formula> ) <an_formula> <annotation>* )
```

Performance Comparison of SMT Solvers

	Module	#L	B	#P	CVC3			Boolector			Z3		
					Size	Time	Failed	Size	Time	Failed	Size	Time	Failed
1	BubbleSort	43	35	17	9031	28.27	0	3011	1.94	0	6057	2.03	0
		43	140	17	146371	MO	1	48791	182.67	0	97722	163.15	0
2	SelectionSort	34	35	17	6982	8.48	0	1955	0.78	0	5134	0.83	0
		34	140	17	108832	MO	1	29885	74.59	0	79369	74.36	0
3	BellmanFord	49	20	33	1076	0.45	0	326	0.27	0	656	0.3	0
4	Prim	79	8	30	4008	16.88	0	1296	0.5	0	3017	0.48	0
5	StrCmp	14	1000	6	9005	9.88	0	3003	91.145	0	7006	38.75	0
6	SumArray	12	1000	7	3001	1.22	0	1001	0.93	0	2003	4.74	0
7	MinMax	19	1000	9	17989	MO	1	5997	947.58	0	11994	6.22	0
8	InsertionSort	86	35	17	9337	35.57	0	3113	2.37	0	6328	2.51	0
		86	140	17	147622	MO	1	49208	TO	1	98833	143	0
9	Fibonacci	83	15	4	16	15.12	0	16	15.6	0	16	15.2	0
10	bs	95	15	7	17	0.21	0	17	0.02	0	17	0.02	0
11	lms	258	202	23	14810	1011.92	0	5005	138.74	0	10211	138.6	0
12	Cubic	66	5	5	40	0.01	0	20	0.19	0	33	0.2	0
13	BitWise	18	8	1	77	272.38	0	27	7.51	0	53	28.37	0
14	adpcm_encode	149	41	12	6417	211.81	0	2377	738.86	0	4878	5.49	0
15	adpcm_decode	111	41	10	23885	43.77	0	9121	20.16	0	19270	14.31	0

Table 1. Results of the comparison between CVC3, Boolector and Z3. Time-outs are represented with TO in the Time column; Examples that exceed available memory are represented with MO in the Time column.

Quoted from "SMT-Based Bounded Model Checking for Embedded ANSI-C Software" by L. Cordeiro, et al ASE 2009

Performance Comparison between CBMC and SMT-CBMC

	Module	#L	B	#P	CBMC						ESW-CBMC					
					Time			#P			Time			#P		
					Encoding	Decision Procedure	Total	Passed	Violated	Failed	Encoding	Decision Procedure	Total	Passed	Violated	Failed
1	sensor	603	5	167	2.04	0.002	2.04	167	0	0	1.23	0.02	1.26	167	0	0
2	crc	125	257	18	5.60	0.003	5.60	18	0	0	4.08	0.07	4.16	18	0	0
3	fft1	218	9	72	0.44	0.001	0.44	72	0	0	0.43	0.005	0.43	72	0	0
4	fft1k	155	1025	39	MO	MO	MO	0	0	39	2337.83	0.055	2337.88	39	0	0
5	fibcall	83	30	2	0.19	0	0.19	2	0	0	0.15	0.002	0.15	2	0	0
6	fir	314	34	25	4.88	0.02	4.9	25	0	0	3.36	0.68	4.04	25	0	0
7	insertsort	86	10	17	0.36	0.005	0.37	17	0	0	0.31	0.02	0.32	17	0	0
8	jfdctint	374	65	331	1.22	0.001	1.22	330	1	0	0.45	2.41	2.86	330	1	0
9	lms	258	202	35	MO	MO	MO	0	0	35	132.6	0.24	132.84	35	0	0
10	ludcmp	144	7	88	4.52	TO	TO	87	0	1	0.017	1.44	1.46	88	0	0
11	matmul	81	6	31	1.16	0	1.16	31	0	0	1.06	0.012	1.07	31	0	0
12	qurt	164	20	8	18.83	TO	TO	7	0	1	1.22	7.7	8.92	8	0	0
13	bcnt	86	17	162	4.42	0.05	4.47	162	0	0	1.24	0.89	2.13	162	0	0
14	blit	95	1	129	0.21	0.001	0.21	128	1	0	0.13	0.28	0.41	128	1	0
15	pocsag	521	42	183	15.32	0.1	15.42	182	1	0	12.33	5.77	18.1	182	1	0
16	adpcm	473	100	553	74.34	3.52	77.86	553	0	0	45.73	9.24	54.97	553	0	0
17	laplace	110	11	76	30.81	TO	TO	0	0	76	12.32	0.29	12.62	76	0	0
18	exStbKey	558	20	18	1.23	0.002	1.23	18	0	0	1.22	0.004	1.23	18	0	0
19	exStbHDMI	1045	15	25	167.91	78.97	246.88	25	0	0	164.43	33.53	197.96	25	0	0
20	exStbLED	430	40	6	195.97	129.8	325.77	6	0	0	165.63	44.53	210.16	6	0	0
21	exStbHwAcc	1432	1000	113	0.67	0.002	0.67	113	0	0	0.72	0.004	0.73	113	0	0
22	exStbResolution	353	200	40	271.8	319.13	590.93	40	0	0	269.31	1161.16	1430.47	40	0	0