Software Model Checking

Equivalence Semantics of CCS

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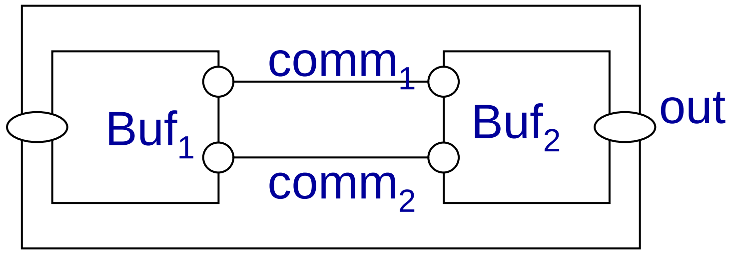
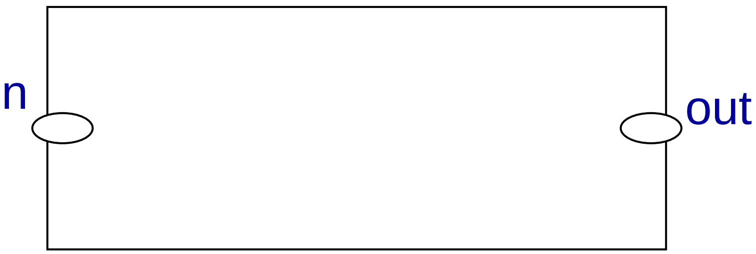
- Trace Equivalence
- Observational Trace Equivalence
- Bisimulation Equivalence
- Observational Bisimulation Equivalence
- Example
- Usage of Concurrent Workbench

- Sys is a design for buffer with separated input/output ports
  - Sys= $(Buf_1 \mid Buf_2)\backslash\{comm_1,comm_2\}$
    - Buf_1 = $in.comm_1'.Buf_1'$, Buf_1' = $comm_2.Buf_1$
    - Buf_2 = $comm_1.Buf_2'$, Buf_2'= $out'.comm_2'.Buf_2$
- Spec is a requirement for the buffer design
  - Spec = in.Spec', Spec'=out'.Spec

Sys= (Buf1 | Buf2)\{$comm_1$,$comm_2$}



Spec= in.out.Spec

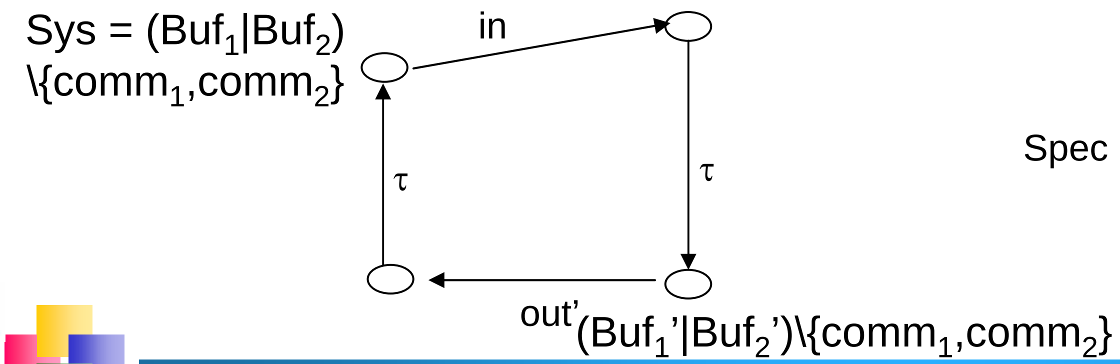


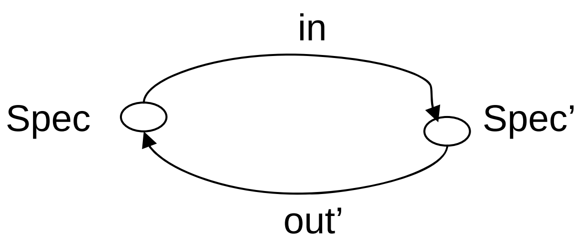
- Question: Sys == Spec?
  - Let us consider trace equivalence (i.e. language equivalence) $=_T$
    - $T(P) = \{ s \in Act^* \mid s$ is an execution trace of P}
    - $P =_T Q$ iff $T(P) = T(Q)$

Observational Trace Equivalence

- Sys $=_T$ Spec?
  - No. Sys has τ which Spec does not
    - T(Sys) = {in, in.τ, in.τ.out' , in.τ.out'.τ,…}
    - T(Spec) = {in , in.out' …}
  - Yes. τ is an internal hidden action not visible outside (not observable). Thus, τ should not be included in an execution
    - If s∈Act*, then ŝ ∈(Act –{τ})* is the action sequence obtained by deleting all occurrences of τ from s.
      - Ex> s = a.τ.b.τ.c, then ŝ = a.b.c
    - A set of observable execution traces: T'(P) = {ŝ | s ∈ T(P)}
    - P $=_{OT}$ Q iff T'(P) = T'(Q)
    - Sys $=_{OT}$ Spec because T'(Sys) = {in, in.out',…}, T'(Spec) = {in, in.out', …}

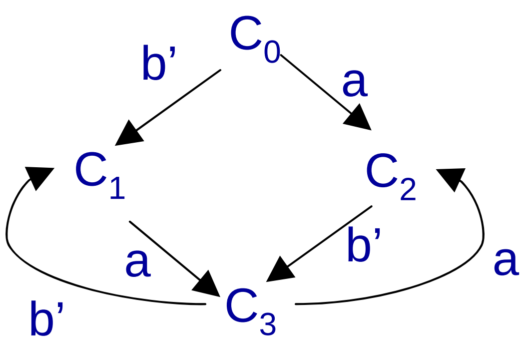
Sys= (Buf1 | Buf2)\{comm1,comm2}
Buf1 = in.comm1.Buf1', Buf1' = comm2.Buf1
Buf2 = comm1'.Buf2',Buf2'=out.comm2'.Buf2
Spec = in.out.Spec

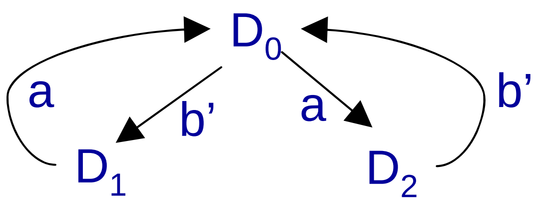




Bisimulation Equivalence

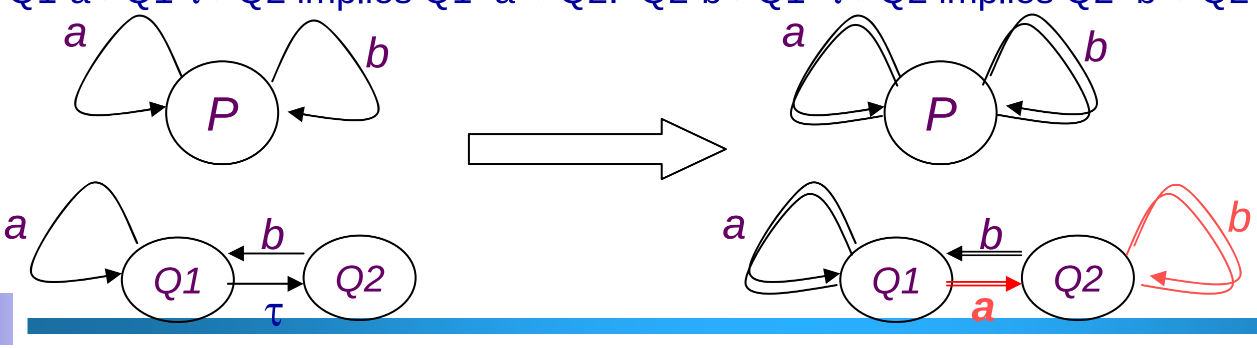
- $P =_{BS} Q$ iff for all $\alpha \in Act$
  - Whenever $P \text{-}\alpha\text{->} P'$, then for some Q', $Q \text{-}\alpha\text{->} Q'$ and $P' =_{BS} Q'$
  - Whenever $Q \text{-}\alpha\text{->} Q'$, then for some P', $P \text{-}\alpha\text{->} P'$ and $P' =_{BS} Q'$
- Note
  - $=_{BS}$ is an equivalence relation (reflexive, transitive, symmetric)
  - $P =_{BS} Q$ implies $P =_T Q$, but not vice versa
- Example>
  - $C_0 = b'.C_1+a.C_2$, $C_1=a.C_3$, $C_2=b'.C_3$, $C_3 = b'.C_1+a.C_2$
  - $D_0 = b'.D_1 +a.D_2$, $D_1=a.D_0$, $D_2=b'.D_0$
  - A binary relation R proves that $C_0 =_{BS} D_0$
    - $R = \{(C_0,D_0), (C_1,D_1), (C_2,D_2), (C_3,D_0)\}$





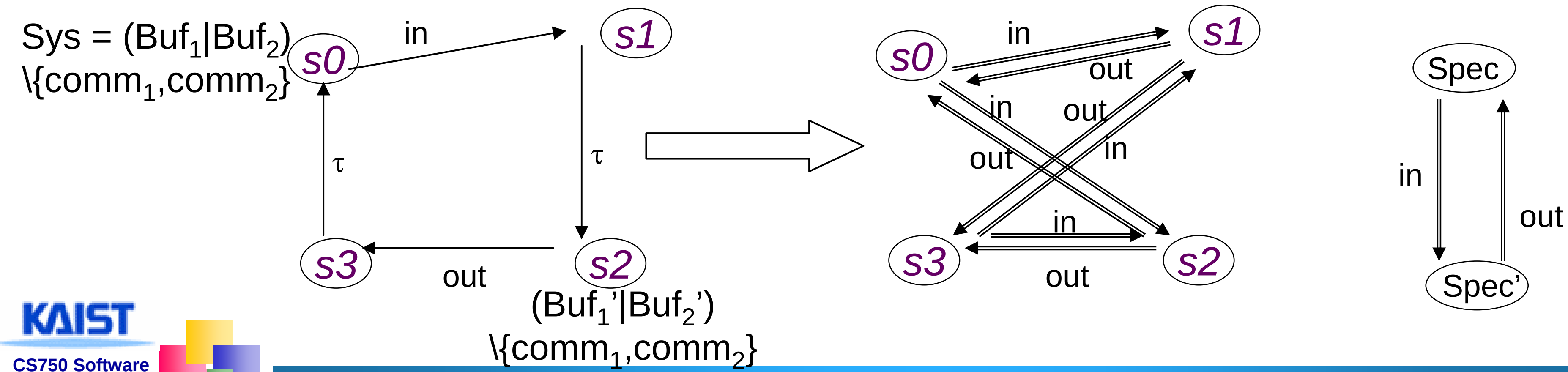
Observational Bisimulation Equivalence

- We cannot simply ignore τ for observational bisimulation equivalence. Thus, we define a new observational transition $=\alpha=>$
- P $=_{OBS}$ Q iff for all $\alpha \in Act$
  - Whenever P =α=> P', then for some Q', Q =α=> Q' and P' $=_{OBS}$ Q'
  - Whenever Q =α=> Q', then for some P', P =α=> P' and P' $=_{OBS}$ Q'
- P =α=> Q iff P (-τ->)*-α->(-τ->)* Q where $\alpha \in Act-\{\tau\}$
  - Let s$\in$(Act-{τ})*. Then q =s=> q' if there exists s' s.t. q-s'->q' and s=ŝ'
  - P = a.P + b.P, Q1=a.Q1 + τ.Q2, Q2=b.Q1
    - Suppose that 'a' means pushing button 'a'. Similarly for 'b'
      - P always allows a user to push any buttons.
      - Q1 allows a user to push button 'a' sometimes, button 'b' sometimes.
    - Thus, we need to distinguish P from Q1 (P and Q1 are not observationally bisimilar), which can be done using =α=> instead of -α->
      - Q1-a->Q1 implies Q1=a=>Q1. Similary Q2-b->Q1 implies Q2=b=>Q1
      - Q1-a->Q1-τ->Q2 implies Q1=a=>Q2. Q2-b->Q1- τ->Q2 implies Q2=b=>Q2



Observational Bisimulation Equivalence (cont)

- Sys $=_{BS}$ Spec? (see slide 3)
  - No. Sys has τ which Spec does not (i.e. not strongly bisimilar)
- Sys $=_{OBS}$ Spec?
  - Yes. Sys is observationally bismilar to Spec
    - Proof: R = { (s0,Spec), (s1,Spec'),(s3,Spec),(s2,Spec')}
      - s0 –in->s1 implies s0=in=> s1. Similarly, s2-out->s3 implies s2=out=>s3
      - s0 -in->s1 -τ->s2 implies s0=in=>s2.
      - s2-out->s3-τ-> s0 implies s2=out=>s0



CWB-NC Commands

- load <ccs filename>
- help <command>
- ls
- cat <process>
- compile <process>
- es <script file> <output file>
- eq –S <trace|bisim|obseq> <proc1> <proc2>
- le –S may <proc1> <proc2> /* Trace subset relation */
- sim <process>
  - semantics <bisim|obseq>
  - random <n>
  - back <n>
  - break <act list>
  - history
  - quit
- quit

Example: Faulty Mutual Exclusion Protocol

```
byte cnt, byte x,y,z;
active[2] proctype user()
{       byte me = _pid +1; /* me is 1 or 2*/
again:
        x = me;
        If
        :: (y ==0 || y== me) -> skip
        :: else -> goto again
        fi;

        z =me;
        If
        :: (x == me) -> skip
        :: else -> goto again
        fi;

        y=me;
        If
        :: (z==me) -> skip
        :: else -> goto again
        fi;

        /* enter critical section */
        cnt++
        assert( cnt ==1);
        cnt --;
        goto again
}
```

```
proc Sys = (P1|P2|X0|Y0|Z0|CNT0)\{x_[0-2],y_[0-2],z_[0-2],
test_x_[0-2],test_y_[0-2],test_z_[0-2], inc_cnt,dec_cnt}

proc P1    = x_1.(test_y_0.P1' + test_y_1.P1' + test_y_2.P1)
proc P1'   = z_1.(test_x_0.P1  + test_x_1.P1'' + test_x_2.P1)
proc P1''  = y_1.(test_z_0.P1  + test_z_1.P1''' + test_z_2.P1)
proc P1''' = inc_cnt.dec_cnt.P1

proc P2    = x_2.(test_y_0.P2' + test_y_1.P2 + test_y_2.P2')
proc P2'   = z_2.(test_x_0.P2  + test_x_1.P2 + test_x_2.P2'')
proc P2''  = y_2.(test_z_0.P2  + test_z_1.P2 + test_z_2.P2''')
proc P2''' = inc_cnt.dec_cnt.P2

* Variable x, y,z, and cnt
proc UpdateX = 'x_0.X0 + 'x_1.X1 + 'x_2.X2
proc X0 = 'test_x_0.X0 + UpdateX
proc X1 = 'test_x_1.X1 + UpdateX
proc X2 = 'test_x_2.X2 + UpdateX

proc UpdateY = 'y_0.Y0 + 'y_1.Y1 + 'y_2.Y2
proc Y0 = 'test_y_0.Y0 + UpdateY
proc Y1 = 'test_y_1.Y1 + UpdateY
proc Y2 = 'test_y_2.Y2 + UpdateY

proc UpdateZ = 'z_0.Z0 + 'z_1.Z1 + 'z_2.Z2
proc Z0 = 'test_z_0.Z0 + UpdateZ
proc Z1 = 'test_z_1.Z1 + UpdateZ
proc Z2 = 'test_z_2.Z2 + UpdateZ

proc CNT0 = 'inc_cnt.cnt_1.CNT1
proc CNT1 = 'inc_cnt.cnt_2.CNT2 + 'dec_cnt.cnt_0.CNT0
proc CNT2 = 'dec_cnt.cnt_1.CNT1

proc Spec = cnt_1.cnt_0.Spec
```

Homework #1: Due Sep 21

- Draw LTS diagrams of Sys (slide 10 of lecture 3) with proofs for all transitions. Also specify which two actions make τ if any.
- Minimize Sys specification (faulty mutual exclusion in the previous slide) by using relabelling functions
- Specify Peterson's mutual exclusion protocol for 2 processes and verify its correctness using CWB-NC

```
/* Peterson's solution to the mutual exclusion problem - 1981 */
boolean turn, flag[2];
byte ncrit;
active [2] proctype user(){
again:   flag[_pid] = 1;
         turn = _pid;
         while(!(flag[1 - _pid] == 0 || turn == 1 - _pid));

         ncrit++;
         assert(ncrit == 1);        /* critical section */
         ncrit--;

         flag[_pid] = 0;
         goto again;
}
```