2016 Fall CS350 HW1 Solution

1. Write corresponding 1st order logic formulas. Explain the meaning of each symbol (i.e., relation, function, and constants) you use

1. Bill has at least one sister.
2. Bill has no sister.
3. Bill has at most one sister.
4. Bill has exactly one sister.
5. Bill has at least two sisters.
6. Every student takes at least one course.
7. Only one student failed History.
8. No student failed Chemistry but at least one student failed History.
9. Every student who takes Analysis also takes Geometry.
10. No student can fool all the other students.

1. $\exists x\ IsSisterOf(x,\ Bill)$
   where IsSisterOf(x, y) means that x is sister of y.
2. $\neg\ \exists x\ IsSisterOf(x,\ Bill)$
3. $\forall x, y\ IsSisterOf(x,\ Bill) \wedge IsSisterOf(y,\ Bill) \rightarrow x = y$
4. $\exists x\ (IsSisterOf(x,\ Bill) \wedge \forall y(IsSisterOf(y,\ Bill) \rightarrow x = y))$
5. $\exists x \exists y\ (IsSisterOf(x,\ Bill) \wedge IsSisterOf(y,\ Bill)\ \wedge\ \neg\ (x = y))$
6. $\forall x\ \Big(Student(x) \rightarrow \exists y\ \big(Course(y) \wedge Takes(x,\ y)\big)\Big)$
   where Student(x) means that x is a student, Course(x) means that x is a course, and Takes(x, y) means that x takes y.
7. $\exists x(Student(x) \wedge Failed(x,\ History) \wedge \forall y(Student(y) \wedge Failed(y,\ History) \rightarrow x = y))$
   where Failed(x, y) means that x failed y.
8. $\forall x(Student(x) \wedge \neg Failed(x,\ Chemistry)) \wedge \exists y(Student(y) \wedge Failed(y,\ History))$
9. $\forall x(Student(x) \wedge Takes(x,\ Analysis) \rightarrow Takes(x,\ Geometry))$
10. $\neg\ \exists x\ (\ Student(x) \wedge\ \forall y\ \ (Student(y) \wedge\ \ \neg\ (x = y) \rightarrow Fools(x,y)\ ))$ where Fools(x, y) means that x can fool y.

2. **Requirement on Retail Chain Management SW**

If the sales for the current month are below the target sales, then a report is to be printed,

- unless the difference between target sales and actual sales is less than half of the difference between target sales and actual sales in the previous month
- or if the difference between target sales and actual sales for the current month is under 5 percent.

Assumption

1. If a current month is the initial month, we don't care about the condition on the previous month
2. The retail chain management SW started to operate on Jan 2016
3. For earlier months than the initial month, no report should be printed.

4. "**the difference** between target sales and actual sales" indicates target sales – actual sales.
5. "under **5 percent**" means 5% of the target sales in a current month.

I = <D, R, F, C> where

- *D =* { *(m, ts, as, r)*|
  m is a month such as Jan-2016,
  ts ∈ R^+ is an amount of target sales in Korean won,
  as ∈ $R^+ \cup \{0\}$ is an amount of actual sales in Korean won,
  r ∈ $\{true, false\}$ is a flag to report or not,
  and if *m1 = m2* for *(m1,ts1,as1,r1)* and *(m2,ts2,as2,r2)*, *ts1=ts2, as1=as2*, and *r1=r2*
  (i.e., each month has unique target sales, actual sales, and report data }

- R = { *is-earlier, is-later*} where
  Is-earlier (m1, m2) return true iff *m1* is earlier than *m2*
  Is-later(m1, m2) return true iff *m1* is later than *m2*

- F = { *month, target_sales, actual_sales, report, prev*} where
  month(m,ts,as,r) returns *m*,
  t_sales (m,ts,as,r) returns *ts*,
  a_sales (m,ts,as,r) returns *as*,
  report (m,ts,as,r) returns *r* and
  prev (m,ts,as,r) returns *(m', ts', as', r')* where *m'* is a previous month of *m*

- C = { m_{init} } where m_{init} indicates the month when the retail chain store started to operate, which is Jan-2016 (see the assumption)

∀d∈*D* (f1 ∨ f2 ∨ f3) where

f1= is-earlier(month(d), m_{init}) ∧ report(d) = false,

f2= (month(d)=m_{init} ∧ a_sales(d)<t_sales(d) ∧ ¬ (t_sales(d)–a_sales(d) < 0.05 x t_sales(d))
) → report(d) = true,

f3=(is-later(month(d), m_{init}) ∧ (a_sales(d) < t_sales(d)) ∧
¬ ((t_sales(d)–a_sales(d) < 0.5 x (t_sales(prev(d))–a_sales(prev(d)))) ∨
(t_sales(d)–a_sales(d) < 0.05 x t_sales(d))
)
) → report(d) = true

2.1 <{ (Jan-2016, 100, 94, true)}, R, F, C >

2.2 <{ (Jan-2016, 100, 94, true), (Feb-2016, 100, 90, false)}, R, F, C >