HW #4: Due Dec 5th 11:00 AM

1. Prove the following sequents in predicate logic:

(a) $\forall x\,\forall y\,\forall z\,(S(x,y) \wedge S(y,z) \rightarrow S(x,z)),\ \forall x\,\neg S(x,x)$
$\vdash \forall x\,\forall y\,(S(x,y) \rightarrow \neg S(y,x))$

(b) $\forall x\,(P(x) \vee Q(x)),\ \exists x\,\neg Q(x),\ \forall x\,(R(x) \rightarrow \neg P(x)) \vdash \exists x\,\neg R(x)$

(c) $\forall x\,(P(x) \rightarrow (Q(x) \vee R(x))),\ \neg\exists x\,(P(x) \wedge R(x)) \vdash \forall x\,(P(x) \rightarrow Q(x))$

(d) $\exists x\,\exists y\,(S(x,y) \vee S(y,x)) \vdash \exists x\,\exists y\,S(x,y)$

(e) $\exists x\,(P(x) \wedge Q(x)),\ \forall y\,(P(x) \rightarrow R(x)) \vdash \exists x\,(R(x) \wedge Q(x))$.

2. Prove the following sequents in predicate logic **and semantic tableau**

(a) $S \rightarrow \exists x\, Q(x) \vdash \exists x\, (S \rightarrow Q(x))$

(b) $\exists x\, P(x) \rightarrow S \vdash \forall x\, (P(x) \rightarrow S)$

(c) $\forall x\, P(x) \rightarrow S \vdash \exists x\, (P(x) \rightarrow S)$

(d) $\forall x\, (P(x) \vee Q(x)) \vdash \forall x\, P(x) \vee \exists x\, Q(x)$

(e) $\forall x\, \exists y\, (P(x) \vee Q(y)) \vdash \exists y\, \forall x\, (P(x) \vee Q(y))$