

**HW#2. Due Oct 17th (Wed 11AM)**

1. Prove in  $\mathcal{G}$  and  $\mathcal{H}$

$$\begin{aligned} &\vdash (A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A), \\ &\vdash (A \rightarrow B) \rightarrow ((\neg A \rightarrow B) \rightarrow B), \\ &\vdash ((A \rightarrow B) \rightarrow A) \rightarrow A. \end{aligned}$$

2. Prove that if  $\vdash U$  in  $\mathcal{G}$  then there is a closed semantic tableau for  $\bar{U}$  (the forward direction of Theorem 3.6).

3. Prove the derived rule called *modus tollens*:

$$\frac{\vdash \neg B \qquad \vdash A \rightarrow B}{\vdash \neg A}.$$

4. Give proofs in  $\mathcal{G}$  for each of the three axioms of  $\mathcal{H}$ .

5. Prove  $\vdash (\neg A \rightarrow A) \rightarrow A$  (Theorem 3.29).