The Satisfiability Modulo Theories Library (SMT-LIB)

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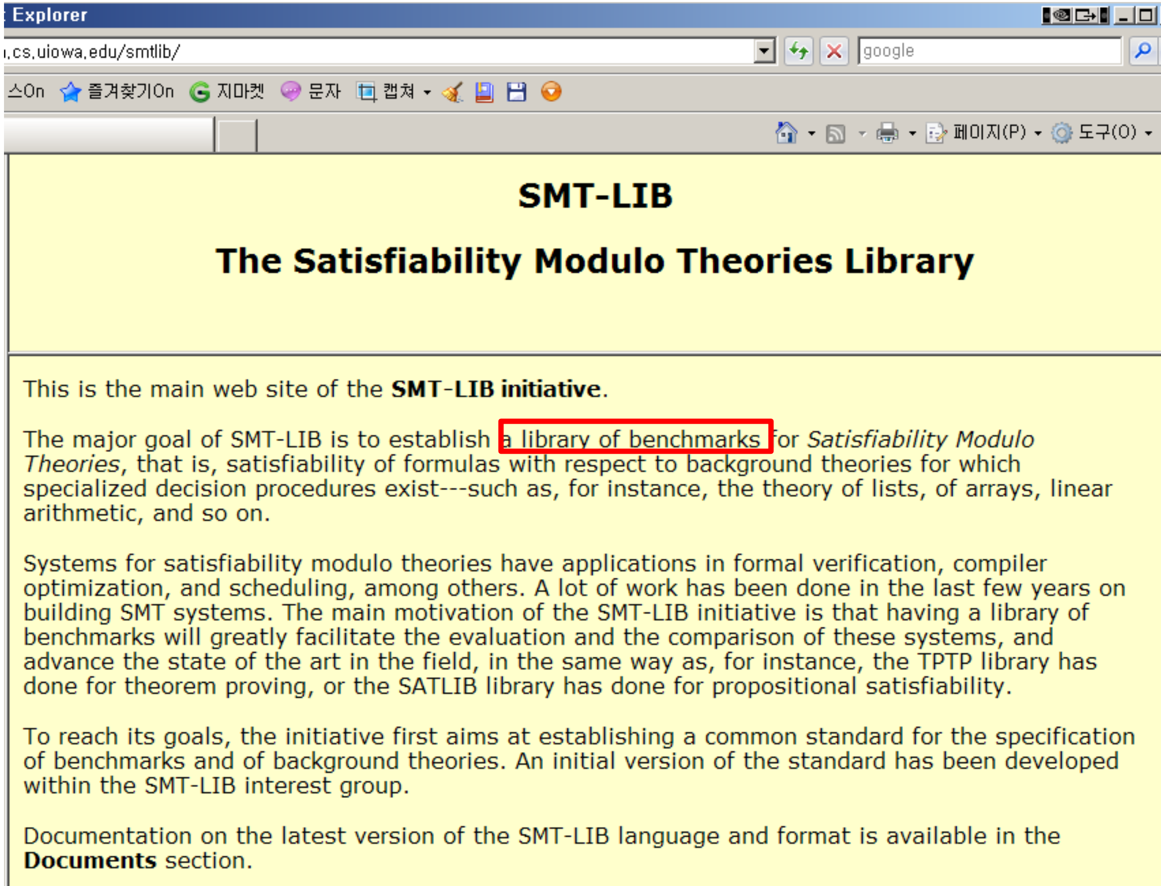


Supported Theories (SMT-Lib v2)

- **ArraysEx**
  - Functional arrays with extensionality.
- **Fixed_Size_BitVectors**
  - Bit vectors with arbitrary size.
- **Core**
  - Core theory, defining the basic Boolean operators
- **Ints**
  - Integer numbers.
- **Reals**
  - Real numbers.
- **Reals_Ints**
  - Real and integer numbers.

Supported Sublogics

AUFLIA: Closed formulas over the theory of linear integer arithmetic and arrays extended with free sort and function symbols but restricted to arrays with integer indices and values.
AUFLIRA: Closed linear formulas with free sort and function symbols over one- and two-dimentional arrays of integer index and real value.
AUFNIRA: Closed formulas with free function and predicate symbols over a theory of arrays of arrays of integer index and real value.
LRA: Closed linear formulas in linear real arithmetic.
QF_ABV: Closed quantifier-free formulas over the theory of bitvectors and bitvector arrays.
QF_AUFBV: Closed quantifier-free formulas over the theory of bitvectors and bitvector arrays extended with free sort and function symbols.
QF_AUFLIA: Closed quantifier-free linear formulas over the theory of integer arrays extended with free sort and function symbols.
QF_AX: Closed quantifier-free formulas over the theory of arrays with extensionality.
QF_BV: Closed quantifier-free formulas over the theory of fixed-size bitvectors.
QF_IDL: Difference Logic over the integers. In essence, Boolean combinations of inequations of the form $x - y < b$ where x and y are integer variables and b is an integer constant.
QF_LIA: Unquantified linear integer arithmetic. In essence, Boolean combinations of inequations between linear polynomials over integer variables.
QF_LRA: Unquantified linear real arithmetic. In essence, Boolean combinations of inequations between linear polynomials over real variables.
QF_NIA: Quantifier-free integer arithmetic.
QF_NRA: Quantifier-free real arithmetic.
QF_RDL: Difference Logic over the reals. In essence, Boolean combinations of inequations of the form $x - y < b$ where x and y are real variables and b is a rational constant.
QF_UF: Unquantified formulas built over a signature of uninterpreted (i.e., free) sort and function symbols.
QF_UFBV: Unquantified formulas over bitvectors with uninterpreted sort function and symbols.
QF_UFIDL: Difference Logic over the integers (in essence) but with uninterpreted sort and function symbols.
QF_UFLIA: Unquantified linear integer arithmetic with uninterpreted sort and function symbols.
QF_UFLRA: Unquantified linear real arithmetic with uninterpreted sort and function symbols.
QF_UFNRA: Unquantified non-linear real arithmetic with uninterpreted sort and function symbols.
UFLRA: Non-linear real arithmetic with uninterpreted sort and function symbols.
UFNIA: Non-linear integer arithmetic with uninterpreted sort and function symbols.

Theory of Arrays

```
(theory Arrays
 :written_by {Silvio Ranise and Cesare Tinelli}
 :date {08/04/05}
 :sorts (Index Element Array)
 :funs ((select Array Index Element)
        (store Array Index Element Array))

 :definition
 "This is a theory of functional arrays without extensionality.
  It is formally and completely defined by the axioms below. "

 :axioms (
  (forall (?a Array) (?i Index) (?e Element)
    (= (select (store ?a ?i ?e) ?i) ?e))

  (forall (?a Array) (?i Index) (?j Index) (?e Element)
    (or (= ?i ?j)
      (= (select (store ?a ?i ?e) ?j)  (select ?a ?j))))
 )
```

Predefined data types (→ :sorts)

Predefined functions (→ store)

Bounded variables (→ ?a)

Prefix operator (→ =)

```
:notes
"It is not difficult to prove that the two
    axioms above are logically equivalent
    to the following \"McCarthy axiom\":

(forall  (?a Array) (?i Index) (?j Index)
         (?e Element)
       (= (select (store ?a ?i ?e) ?j)
          (ite (= ?i ?j) ?e (select ?a ?j))))

Such an axiom appeared in the following
    paper:
Correctness of a Compiler for Arithmetic
    Expressions,
by John McCarthy and James Painter,
available at http://www-formal.stanford.
    edu/jmc/mcpain.html.
"
)
```

If-then-else term construct (→ ite)

Theory of Arrays w/ Extensionability

```
(theory ArraysEx
 :written_by {Silvio Ranise and Cesare Tinelli}
 :date {08/04/05}
 :updated {28/10/05}
 :history {
  Bug fix in the third axiom, pointed out by Robert
  Nieuwenhuis:
  The scope of 'forall (?i Index)' was the whole implication
  instead of just the premise of the implication.
  }
 :sorts (Index Element Array)
 :funs ((select Array Index Element)
          (store Array Index Element Array))
 :definition
 "This is a theory of functional arrays with extensionality.
  It is formally and completely defined by the axioms below.
 "
```

```
 :axioms
 (
  (forall (?a Array) (?i Index) (?e Element)
    (= (select (store ?a ?i ?e) ?i) ?e))
  (forall (?a Array) (?i Index) (?j Index) (?e Element)
    (or (= ?i ?j)
        (= (select (store ?a ?i ?e) ?j)
           (select ?a ?j))))

  (forall (?a Array) (?b Array)
     (implies (forall (?i Index) (= (select ?a ?i) (select ?b ?i)))
                 (= ?a ?b)))
)
:notes "This theory extends the theory Arrays with
 an axiom stating that any two arrays with the same
 elements are in fact the same array. " )
```

Theory of Integer

(theory Ints

:sorts (Int)

:notes

"The (unsupported) annotations of the function/pre
dicate symbols have

the following meaning:

attribute | possible value | meaning

:assoc // the symbol is associative

:comm // the symbol is commutative

:unit a constant

:trans // the symbol is transitive

:refl // the symbol is reflexive

:irref // the symbol is irreflexive

:antisym // the symbol is antisymmetric

"

:funs ((0 Int)

(1 Int)

(~ Int Int) ; unary minus

(- Int Int Int) ; binary minus

(+ Int Int Int :assoc :comm :unit {0})

(* Int Int Int :assoc :comm :unit {1}))

:preds ((<= Int Int :refl :trans :antisym)

(< Int Int :trans :irref)

(>= Int Int :refl :trans :antisym)

(> Int Int :trans :irref)

)

:definition

"This is the first-order theory of the integers, that is , the set of all the first-order sentences over the given signature that are true in the structure of the integer numbers interpreting the signature's symbols in the obvious way (with ~ denoting the negation and - the subtraction functions). "

:notes

"Note that this theory is not (recursively) axiomatizable in a first-order logic such as SMT-LIB's underlying logic. That is why the theory is defined semantically. "

)

Example of QF_LIA Benchmark

```
(benchmark example
:status sat
:logic QF_LIA
:extrafuns ((x1 Int)
(x2 Int) (x3 Int) (x4 Int) (x5 Int))
;human readable form
; x1-1 => x2 /\
; x1-3 <= x2 /\
; x1 = 2 x3+x5 /\
; x3 = x5   /\
; x2 = 6 x4
:formula (and
        (>= (- x1 x2) 1)
        (<= (- x1 x2) 3)
        (= x1 (+ (* 2 x3) x5))
        (= x3 x5)
        (= x2 (* 6 x4))))
```

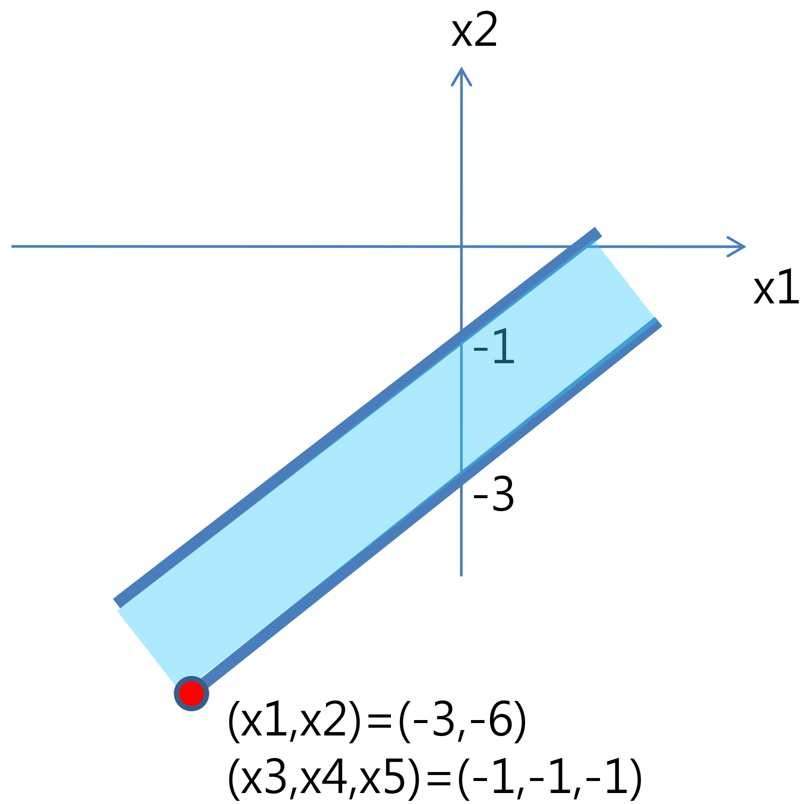
Expected output (optional)

Theory

User defined variables

Comments

Target formula



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Example of QF_UF Benchmark

```
(benchmark example2-1
:logic QF_UF
:extrasorts (A B C D)
:extrafuns  ((x A)(y B)(w A)(z C)(u D))
:extrafuns  ((f A A B)
(g A B B) (h1 B A B) (h2 B B B))

;human readable form
;   g(x,y) = h1(y,x) /\
;   f(x,x) = h2(y,y) /\
;   f(x,x) != f(x,w)
:assumption((= (g x y) (h1 y x)))
:assumption((= (f x x) (h2 y y))
:assumption((not (= (f x x) (f x w))))
:formula true
)
```

User defined data types

User defined functions

<u>A model for the formula</u>

x-> v0

y->v1

w->v4

g->{ (v1,v0)->v2,
else-> v2}

f->{(v0,v0)->v3,
(v0,v4)->v5,
else->v5}

h2->{(v1,v1)->v3,
else -> v3}

Another Example of QF_UF Benchmark

3. Prove that

F : a=b ^ b=c -> g(f(a), b) = g(f (c), a)

is T_{EUF}–**satisfiable**

4. Prove

F : a=b ^ b=c -> g(f(a), b) = g(f (c), a)

is T_{EUF}–**valid** through proof tree

```
(benchmark Quiz2
:logic QF_UF
:extrasorts (A B C)
:extrafuns  ((a A) (b A) (c A))
:extrafuns  ((f A B) (g B A C))
;human readable form
; a=b/\ b=c ->  g(f(a),b) = g(f(c),a)
:formula (implies (and (= a b) (= b c))
              (= (g ( f a) b) (g (f c) a)))))
```

<u>A model for the formula</u>

a->v0

b->v1

c->v2

f->{v0->v3,v2->v5,else->v5}

g->{(v3,v1)->v4,(v5,v0)->v6,else->v6}

Then, how to check T_{EUF}–validity of F by using a SMT solver???

Example of QF_AUFLIA

```
(benchmark sort
:logic QF_AUFLIA
:extrafuns ((data_7 Array)) ; initial data[] declaration
:extrafuns ((tmp_0 Int))
:extrafuns ((i_9 Int))
 :assumption (= i_9 0) ; i=0;
:extrafuns ((j_1 Int))
 :assumption (= j_1 1) ; j=1;
:extrafuns ((tmp_1 Int))
 :assumption (= tmp_1 (select data_7 0))   ; tmp = data[0]
:extrafuns ((data_8 Array))
 :assumption (= data_8 (store data_7 0 (select data_7 1))); data[0]=data[1];
:extrafuns ((data_9 Array))
 :assumption (= data_9 (store data_8 1 tmp_1)) ; data[1] =  tmp;
:extrafuns ((data_10 Array))
 ; if (data[0] > data[1])  { tmp=data[0]; data[0]=data[1]; data[1]=tmp}
 :assumption (= data_10 (if_then_else (> (select data_7 0) (select data_7 1)
      ) data_9 data_7))
…
:formula (not
(and (<= (select data_70 0) (select data_70 1))
   (<= (select data_70 1) (select data_70 2))
     …)
```

```c
#define N 7
int main(){
  int data[N], i, j, tmp;
  for (i=0; i<N-1; i++)
    for (j=i+1; j<N; j++)
       if(data[i]>data[j]){
          tmp = data[i];
          data[i] = data[j];
          data[j] = tmp;
       }
  assert(data[0]<=data[1]&&…);
}
```


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Theory of Fixed_Size_BitVectors[32]

:sorts_description
"All sort symbols of the form BitVec[i],
where i is a numeral between 1 and 32, inclusive."
:funs_description
"All function symbols with arity of the form
(concat BitVec[i] BitVec[j] BitVec[m]) where
- i,j,m are numerals
- i,j > 0
- i + j = m <= 32 "
:funs_description
"All function symbols with arity of the form
(extract[i:j] BitVec[m] BitVec[n]) where
- i,j,m,n are numerals
- 32 >= m > i >= j >= 0,
- n = i-j+1. "
:funs_description
"All function symbols with arity of the form
(op1 BitVec[m] BitVec[m]) or
(op2 BitVec[m] BitVec[m] BitVec[m]) where
- op1 is from {bvnot, bvneg}
- op2 is from {bvand,bvor,bvxor,bvsub,bvadd,bvmul}
- m is a numeral
- 0 < m <= 32 "
:preds_description
"All predicate symbols with arity of the form
(pred BitVec[m] BitVec[m]) where
- pred is from {bvlt, bvleq, bvgeq, bvgt}
- m is a numeral
- 0 < m <= 32 "

- Variables
If v is a variable of sort BitVec[m] with 0 < m <= 32, then
[[v]] is some element of [{0,...,m-1} -> {0,1}], the set of
total functions from {0,...,m-1} to {0,1}.
- Constant symbols bv0 and bv1 of sort BitVec[32]
[[bv0]] := \lambda x : [0...32). 0
[[bv1]] := \lambda x : [0...32). if x = 0 then 1 else 0
- Function symbols for concatenation
[[(concat s t)]] := \lambda x : [0...n+m).
if (x<m) then [[t]](x) else [[s]](x-m) where
s and t are terms of sort BitVec[n] and BitVec[m],
respectively, 0 < n <= 32, 0 < m <= 32, and n+m <= 32.
- Function symbols for extraction
[[(extract[i:j] s)]] := \lambda x : [0...i-j+1). [[s]](j+x)
where s is of sort BitVec[l], 0 <= j <= i < l <= 32.
- Function symbols for arithmetic operations
To define the semantics of the bitvector operators bvadd
, bvsub, bvneg, and bvmul, it is helpful to use these
ancillary functions:
o bv2nat which takes a bitvector b: [0...m) --> {0,1}
with 0 < m <= 32, and returns an integer in the range
[0...2^m), and is defined as follows:
bv2nat(b) := b(m-1)*2^{m-1} + b(m-2)*2^{m-2} + ... + b(0)*2^0
o nat2bv[m], with 0 < m <= 32, which takes a non-negative
integer n and returns the (unique) bitvector b: [0,...,m) -> {0,1}
such that b(m-1)*2^{m-1} + ... + b(0)*2^0 = n MOD 2^m
where MOD is usual modulo operation.
[[(bvadd s t)]] := nat2bv[m](bv2nat(s) + bv2nat(t))

SMTLIB Benchmark Syntax

- Reserved keywords
  - =, and, benchmark, distinct, exists, false, flet, forall, if then el se, iff, implies,ite, let, logic, not, or, sat, theory, true, unknow n, unsat, xor

Formulas

⟨*prop_atom*⟩ ::= `true` | `false` | ⟨*fvar*⟩ | ⟨*identifier*⟩

⟨*an_atom*⟩ ::= ⟨*prop_atom*⟩ | (⟨*prop_atom*⟩ ⟨*annotation*⟩$^{+}$)
| (⟨*pred_symb*⟩ ⟨*an_term*⟩$^{+}$ ⟨*annotation*⟩*)

⟨*connective*⟩ ::= `not` | `implies` | `if_then_else`
| `and` | `or` | `xor` | `iff`

⟨*quant_symb*⟩ ::= `exists` | `forall`

⟨*quant_var*⟩ ::= (⟨*var*⟩ ⟨*sort_symb*⟩)

⟨*an_formula*⟩ ::= ⟨*an_atom*⟩
| (⟨*connective*⟩ ⟨*an_formula*⟩$^{+}$ ⟨*annotation*⟩*)
| (⟨*quant_symb*⟩ ⟨*quant_var*⟩$^{+}$ ⟨*an_formula*⟩ ⟨*annotation*⟩*)
| (`let` (⟨*var*⟩ ⟨*an_term*⟩) ⟨*an_formula*⟩ ⟨*annotation*⟩*)
| (`flet` (⟨*fvar*⟩ ⟨*an_formula*⟩) ⟨*an_formula*⟩ ⟨*annotation*⟩*)

Performance Comparison of SMT Solvers

	Module	#L	B	#P	CVC3 Size	CVC3 Time	CVC3 Failed	Boolector Size	Boolector Time	Boolector Failed	Z3 Size	Z3 Time	Z3 Failed
1	BubbleSort	43	35	17	9031	28.27	0	3011	1.94	0	6057	2.03	0
		43	140	17	146371	MO	1	48791	182.67	0	97722	163.15	0
2	SelectionSort	34	35	17	6982	8.48	0	1955	0.78	0	5134	0.83	0
		34	140	17	108832	MO	1	29885	74.59	0	79369	74.36	0
3	BellmanFord	49	20	33	1076	0.45	0	326	0.27	0	656	0.3	0
4	Prim	79	8	30	4008	16.88	0	1296	0.5	0	3017	0.48	0
5	StrCmp	14	1000	6	9005	9.88	0	3003	91.145	0	7006	38.75	0
6	SumArray	12	1000	7	3001	1.22	0	1001	0.93	0	2003	4.74	0
7	MinMax	19	1000	9	17989	MO	1	5997	947.58	0	11994	6.22	0
8	InsertionSort	86	35	17	9337	35.57	0	3113	2.37	0	6328	2.51	0
		86	140	17	147622	MO	1	49208	TO	1	98833	143	0
9	Fibonacci	83	15	4	16	15.12	0	16	15.6	0	16	15.2	0
10	bs	95	15	7	17	0.21	0	17	0.02	0	17	0.02	0
11	lms	258	202	23	14810	1011.92	0	5005	138.74	0	10211	138.6	0
12	Cubic	66	5	5	40	0.01	0	20	0.19	0	33	0.2	0
13	BitWise	18	8	1	77	272.38	0	27	7.51	0	53	28.37	0
14	adpcm_encode	149	41	12	6417	211.81	0	2377	738.86	0	4878	5.49	0
15	adpcm_decode	111	41	10	23885	43.77	0	9121	20.16	0	19270	14.31	0

Table 1. Results of the comparison between CVC3, Boolector and Z3. Time-outs are represented with TO in the Time column; Examples that exceed available memory are represented with MO in the Time column.

Quoted from "SMT-Based Bounded Model Checking for Embedded ANSI-C Software" by L.Cordeiro, et al ASE 2009

Performance Comparison between CBMC and SMT-CBMC

					CBMC						ESW-CBMC					
					Time			#P			Time			#P		
	Module	#L	B	#P	Encoding	Decision Procedure	Total	Passed	Violated	Failed	Encoding	Decision Procedure	Total	Passed	Violated	Failed
1	sensor	603	5	167	2.04	0.002	2.04	167	0	0	1.23	0.02	1.26	167	0	0
2	crc	125	257	18	5.60	0.003	5.60	18	0	0	4.08	0.07	4.16	18	0	0
3	fft1	218	9	72	0.44	0.001	0.44	72	0	0	0.43	0.005	0.43	72	0	0
4	fft1k	155	1025	39	MO	MO	MO	0	0	39	2337.83	0.055	2337.88	39	0	0
5	fibcall	83	30	2	0.19	0	0.19	2	0	0	0.15	0.002	0.15	2	0	0
6	fir	314	34	25	4.88	0.02	4.9	25	0	0	3.36	0.68	4.04	25	0	0
7	insertsort	86	10	17	0.36	0.005	0.37	17	0	0	0.31	0.02	0.32	17	0	0
8	jfdctint	374	65	331	1.22	0.001	1.22	330	1	0	0.45	2.41	2.86	330	1	0
9	lms	258	202	35	MO	MO	MO	0	0	35	132.6	0.24	132.84	35	0	0
10	ludcmp	144	7	88	4.52	TO	TO	87	0	1	0.017	1.44	1.46	88	0	0
11	matmul	81	6	31	1.16	0	1.16	31	0	0	1.06	0.012	1.07	31	0	0
12	qurt	164	20	8	18.83	TO	TO	7	0	1	1.22	7.7	8.92	8	0	0
13	bcnt	86	17	162	4.42	0.05	4.47	162	0	0	1.24	0.89	2.13	162	0	0
14	blit	95	1	129	0.21	0.001	0.21	128	1	0	0.13	0.28	0.41	128	1	0
15	pocsag	521	42	183	15.32	0.1	15.42	182	1	0	12.33	5.77	18.1	182	1	0
16	adpcm	473	100	553	74.34	3.52	77.86	553	0	0	45.73	9.24	54.97	553	0	0
17	laplace	110	11	76	30.81	TO	TO	0	0	76	12.32	0.29	12.62	76	0	0
18	exStbKey	558	20	18	1.23	0.002	1.23	18	0	0	1.22	0.004	1.23	18	0	0
19	exStbHDMI	1045	15	25	167.91	78.97	246.88	25	0	0	164.43	33.53	197.96	25	0	0
20	exStbLED	430	40	6	195.97	129.8	325.77	6	0	0	165.63	44.53	210.16	6	0	0
21	exStbHwAcc	1432	1000	113	0.67	0.002	0.67	113	0	0	0.72	0.004	0.73	113	0	0
22	exStbResolution	353	200	40	271.8	319.13	590.93	40	0	0	269.31	1161.16	1430.47	40	0	0