2007 3102 김윤호

Date . . No.

1. $\vdash (A \to B) \to (\neg B \to \neg A)$.

in g

$A, \neg A, B \qquad B, \neg B, \neg A$

β-rule

$\neg(A \to B), B, \neg A$

α-rule

$\neg(A \to B), \neg\neg B, \neg A$

α-rule

$\neg(A \to B), (\neg B \to \neg A)$

α-rule

$(A \to B) \to (\neg B \to \neg A)$.

in H.

1.	$\{A \to B, \neg B, \neg\neg A\} \vdash \neg\neg A$	assumption
2.	$\{A \to B, \neg B, \neg\neg A\} \vdash A$	double neg 1
3.	$\{A \to B, \neg B, \neg\neg A\} \vdash A \to B$	assumption
4.	$\{A \to B, \neg B, \neg\neg A\} \vdash B$	MP 2.3
5.	$\{A \to B, \neg B, \neg\neg A\} \vdash \neg B$	assumption
6.	$\{A \to B, \neg B, \neg\neg A\} \vdash \neg B \to (B \to \neg\neg B)$	theorem 3.20.
7.	$\{A \to B, \neg B, \neg\neg A\} \vdash B \to \neg\neg B$	MP 5,6
8.	$\{A \to B, \neg B, \neg\neg A\} \vdash \neg\neg B$.	MP 4, 7
9.	$\{A \to B, \neg B\} \vdash \neg\neg A \to \neg\neg B$	deduction 8
10.	$\{A \to B, \neg B\} \vdash \neg B \to \neg A$	contrapositive 9
11.	$\{A \to B, \neg B\} \vdash \neg B$	assumption
12.	$\{A \to B, \neg B\} \vdash \neg A$	MP 11.10
13.	$\{A \to B\} \vdash \neg B \to \neg A$	deduction 12
14.	$\vdash (A \to B) \to (\neg B \to \neg A)$	deduction 13

MooKeuk

$\vdash (A\to B)\to((\neg A\to B)\to B)$

in $\mathcal{G}$

$\neg A, A, B \qquad A, B, \neg B \qquad \neg A, B, \neg B \qquad B, \neg B$

β-rule $\qquad$ β-rule

$\neg(\neg A\to B), A, B \qquad \neg(\neg A\to B), B, \neg B$

$\neg(\neg A\to B), B, \neg(A\to B)$

$\downarrow$ α-rule

$\neg(A\to B), (\neg A\to B)\to B$ $\quad$ α-rule

$\downarrow$

$(A\to B)\to((\neg A\to B)\to B)$

in $\mathcal{H}$

1. $\{A\to B, \neg A\to B, \neg B\} \vdash \neg B$ assumption
2. $\{A\to B, \neg A\to B, \neg B\} \vdash \neg A\to B$ assumption
3. $\{A\to B, \neg A\to B, \neg B\} \vdash (\neg A\to B)\to(\neg B\to\neg\neg A)$ above theorem (1-1)
4. $\{A\to B, \neg A\to B, \neg B\} \vdash \neg B\to\neg\neg A$ MP 2,3
5. $\{A\to B, \neg A\to B, \neg B\} \vdash \neg\neg A$ MP 1,4
6. $\{A\to B, \neg A\to B, \neg B\} \vdash A$ dauble neg 5
7. $\{A\to B, \neg A\to B, \neg B\} \vdash A\to B$ assumption
8. $\{A\to B, \neg A\to B, \neg B\} \vdash B$ MP 6,7
9. $\{A\to B, \neg A\to B\} \vdash \neg B\to B$ deduction 8
10. $\{A\to B, \neg A\to B\} \vdash (\neg B\to B)\to B$ Theorem 3.29
11. $\{A\to B, \neg A\to B\} \vdash B$ MP 9,10
12. $\{A\to B\} \vdash (\neg A\to B)\to B$ deduction 11
13. $\vdash (A\to B)\to((\neg A\to B)\to B)$ deduction 12

$(\neg B \wedge (A\to B)) \to \neg A$ $\neg(\neg B \to \neg(A\to B)) \to \neg A$

Date . . No.

$\vdash ((A\to B)\to A)\to A$

in $\mathcal{G}$

$$\dfrac{\dfrac{\dfrac{A, \neg A, B \qquad A, \neg A}{A\to B,\ A}\ \alpha\text{-rule},\ \beta\text{-rule}}{\neg((A\to B)\to A),\ A}}{((A\to B)\to A)\to A}\ \alpha\text{-rule}$$

in $\mathcal{H}$

1.	$\{(A\to B)\to A, \neg A\} \vdash \neg A$	assumption
2.	$\{(A\to B)\to A, \neg A\} \vdash \neg A \to (A\to B)$	Theorem 3.20
3.	$\{(A\to B)\to A, \neg A\} \vdash (A\to B)$	MP 1.2.
4.	$\{(A\to B)\to A, \neg A\} \vdash (A\to B)\to A$	assumption
5.	$\{(A\to B)\to A, \neg A\} \vdash A$	MP 3.4.
6.	$\{(A\to B)\to A\} \vdash \neg A \to A$	deduction 5.
7.	$\{(A\to B)\to A\} \vdash (\neg A\to A)\to A$	Theorem 3.29.
8.	$\{(A\to B)\to A\} \vdash A$	MP 6.7
9.	$\emptyset \vdash ((A\to B)\to A)\to A$	deduction 8.

2. Prove that if $\vdash U$ in $\mathcal{G}$ then there is a closed semantic tableau for $\bar{U}$

We prove that there is a closed semantic tableau for $\bar{U}$ by induction on h, the height of a proof tree. If $h=0$, then U is an axiom, containing a complementary pair $\{p, \neg p\}$, that is, $\bar{U} = \bar{U}_0 \cup \{p, \neg p\}$. Semantic tableau rules cannot decompose literals. $\{p, \neg p\}$ is in all leaf nodes. So T for $\bar{U}$ is closed.

If $h>0$, then some Gentzen α- or β-rule was used at the root of a proof tree on a formula $A \in U$, that is, $U = U_0 \cup \{A\}$.

i) Case 1: A Gentzen α-rule was used on $A = A_1 \vee A_2$ to produce the node n' labeled $U' = U_0 \cup \{A_1, A_2\}$. The subtree rooted at n' is a proof tree for U' so by the inductive hypothesis, T for $\bar{U}' (= \bar{U}_0 \cup \{\neg A_1, \neg A_2\})$ is closed. Using the semantic tableau α-rule, T for $\bar{U} (= \bar{U}_0 \cup \{\neg(A_1 \vee A_2)\}$ can be constructed and is closed.

Case 2: A Gentzen β-rule was used on $A = A_1 \wedge A_2$ to produce the nodes n' and n'' labeled $U' = U_0 \cup \{A_1\}$ and $U'' = U_0 \cup \{A_2\}$ respectively. By the inductive hypothesis, T' for $\overline{U'}$ and T'' for $\overline{U''}$ are closed. Using the semantic tableau β-rule T for $\overline{U}$ can be constructed and is closed.

3.

$$\dfrac{\vdash \neg B \qquad \dfrac{\vdash A \rightarrow B \qquad \vdash (A\rightarrow B)\rightarrow(\neg B \rightarrow \neg A)}{\vdash \neg B \rightarrow \neg A}}{\vdash \neg A}$$

4. 1. $\vdash A \rightarrow (B \rightarrow A)$

$A, \neg A, \neg B$

$\downarrow$ α-rule

$\neg A, (B \rightarrow A)$

$\downarrow$ α-rule

$A \rightarrow (B \rightarrow A)$.

2. $\vdash ((A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C)))$

$A, \neg A, B, C \qquad B, \neg B, \neg A, C$

β-rule

$\neg A, B, C, \neg(A \rightarrow B) \qquad C, \neg C,$

β-rule

$\neg(B \rightarrow C), \neg(A \rightarrow B), \neg A, C \qquad A, \neg A,$

β-rule

$\neg(A \rightarrow B), \neg(A \rightarrow (B \rightarrow C)), \neg A, C$

$\downarrow$ α-rule

$\neg(A \rightarrow B), \neg(A \rightarrow (B \rightarrow C)), (A \rightarrow C)$

$\downarrow$ α-rule

$((A \rightarrow B) \rightarrow (A \rightarrow C)), \neg(A \rightarrow (B \rightarrow C))$

$\downarrow$ α-rule

$(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$.

3. $\vdash (\neg B \to \neg\neg A) \to (A \to B)$

$B, \neg B, \neg A \qquad\qquad A, \neg A, B$

$\neg(\neg B \to \neg\neg A), \neg A, B$ β-rule

$\downarrow$ α-rule

$\neg(\neg B \to \neg\neg A), A \to B$ α-rule

$\downarrow$

$(\neg B \to \neg\neg A) \to (A \to B)$

5. $\vdash (\neg A \to A) \to A$ in H

1.	$\{\neg A \to A, \neg A\} \vdash \neg A$	assumption
2.	$\{\neg A \to A, \neg A\} \vdash \neg A \to A$	assumption
3.	$\{\neg A \to A, \neg A\} \vdash A$	MP 1.2
4	$\{\neg A \to A, \neg A\} \vdash A \to (\neg A \to \text{false})$	theorem 3.21
5.	$\{\neg A \to A, \neg A\} \vdash \neg A \to \text{false}$	Mp 3.4
6.	$\{\neg A \to A, \neg A\} \vdash \text{false}$	MP 1, 5
7.	$\{\neg A \to A\} \vdash \neg A \to \text{false}$	deduction 6.
8.	$\{\neg A \to A\} \vdash A$	RAA 7.
9.	$\vdash (\neg A \to A) \to A$	deduction 8