# Introduction to Software Testing Chapter 3.2 Logic Coverage 

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## Covering Logic Expressions

- Logic expressions show up in many situations
- Covering logic expressions is required by the US Federal Aviation Administration for safety critical software
- Logical expressions can come from many sources
- Decisions in programs
- FSMs and statecharts
- Requirements
- Tests are intended to choose some subset of the total number of truth assignments to the expressions


## Logic Coverage Criteria Subsumption



## Logic Predicates and Clauses

- A predicate is an expression that evaluates to a boolean value
- Predicates can contain
- boolean variables
- non-boolean variables that contain >, <, ==, >=, <=, !=
- boolean function calls
- Internal structure is created by logical operators
- $\neg$ - the negation operator
- $\wedge$ - the and operator
- $\vee$ - the or operator
- $\rightarrow$ - the implication operator
$\square \oplus$ - the exclusive or operator
- $\leftrightarrow$ - the equivalence operator
- A clause is a predicate with no logical operators


## Examples

- ( $a<b) \vee f(z) \wedge D \wedge\left(m>=n^{*} 0\right)$
- Four clauses:
- ( $\mathrm{a}<\mathrm{b}$ ) - relational expression
- $f(z)$ - boolean-valued function
- D - boolean variable
- (m >= $n^{*} 0$ ) - relational expression
- Most predicates have few clauses
- Sources of predicates
- Decisions in programs
- Guards in finite state machines
- Decisions in UML activity graphs
- Requirements, both formal and informal
- SQL queries


## Testing and Covering Predicates

- We use predicates in testing as follows :
- Developing a model of the software as one or more predicates
- Requiring tests to satisfy some combination of clauses
- Abbreviations:
- $P$ is the set of predicates
- $p$ is a single predicate in $P$
- $C$ is the set of clauses in $P$
- $C_{p}$ is the set of clauses in predicate $p$
- $c$ is a single clause in $C$


## Predicate and Clause Coverage

- The first (and simplest) two criteria require that each predicate and each clause be evaluated to both true and false
> : For each $p$ in P, TR contains two requirements: $p$ evaluates to true, and $p$ evaluates to false.
a.k.a. "decision coverage" in literature
- When predicates come from conditions on edges, this is equivalent to edge coverage
- PC does not evaluate all the clauses, so ...
$\square$
Coverage (CC) : For each c in C, TR contains two requirements: $c$ evaluates to true, and $c$ evaluates to false.

KAIST a.k.a. "condition coverage" in literature

## Predicate Coverage Example $((a<b) \vee D) \wedge(m>=n *)$

 predicate coveragePredicate = true<br>$a=5, b=10, D=$ true, $m=1, n=1, o=1$<br>$=(5<10) \vee$ true $\wedge(1>=1 * 1)$<br>$=$ true $\vee$ true $\wedge$ TRUE<br>= true

$\quad$| Predicate $=$ false |
| :--- |
| $a=10, b=5, D=$ false, $m=1, n=1, o=1$ |
| $=(10<5) \vee$ false $\wedge(1>=1 * 1)$ |
| $=$ false $\vee$ false $\wedge T R U E$ |
| $=$ false |

## Clause Coverage Example $((a<b) \vee D) \wedge(m>=n *)$

Clause coverage


## Problems with PC and CC

- PC does not fully exercise all the clauses, especially in the presence of short circuit evaluation
- CC does not always ensure PC
- That is, we can satisfy CC without causing the predicate to be both true and false

Ex. $x>3 \rightarrow x>1$

- Two test cases $\{x=4, x=0\}$ satisfy $C C$ but not $P C$
- This is definitely not what we want !
- Condition/decision coverage is a hybrid metric composed by the union of CC and PC
- Modified condition/decision coverage (MC/DC) checks every condition can affect decision
- equivalent to condition/decision coverage for C/Java (w/ short circuit)
- The simplest solution is to test all combinations ...


## Combinatorial Coverage

- CoC requires every possible combination
- Sometimes called Multiple Condition Coverage
Com requirements for the clauses in $C_{p}$ to evaluate to
test req
each possible combination of truth values.

|  | $\mathbf{a}<\mathbf{b}$ | D | $\mathrm{m}>=\mathbf{n}^{*} \mathbf{0}$ | $(\mathbf{( a < b}) \vee \mathrm{D}) \wedge\left(\mathrm{m}>=\mathbf{n}^{*} \mathbf{0}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | T | T | T | T |
| 2 | T | T | F | F |
| 3 | T | F | T | T |
| 4 | T | F | F | F |
| 5 | F | T | T | T |
| $\mathbf{6}$ | F | T | F | F |
| 7 | F | F | T | F |
| $\mathbf{8}$ | F | F | F | F |

## Combinatorial Coverage

- This is simple, neat, clean, and comprehensive ...
- But quite expensive!
- $2^{N}$ tests, where $N$ is the number of clauses
- Impractical for predicates with more than 3 or 4 clauses
- The literature has lots of suggestions - some confusing
- The general idea is simple:


## Test each clause independently from the other clauses

- Getting the details right is hard
- What exactly does "independently" mean?
- The book presents this idea as "making clauses active"


## Active Clauses

- Clause coverage has a weakness
- The values do not always make a difference to a whole predicate
- To really test the results of a clause, the clause should be the determining factor in the value of the predicate


## Determination :

A clause $\boldsymbol{C}_{\boldsymbol{i}}$ in predicate $\boldsymbol{p}$, called the major clause, determines $\boldsymbol{p}$ if and only if the values of the
remaining minor clauses $\boldsymbol{C}_{\boldsymbol{j}}$ are such that changing $C_{i}$ changes the value of $p$

- This is considered to make the clause $\mathrm{c}_{\mathrm{i}}$ active


## Determining Predicates

$$
\begin{aligned}
& \quad \mathrm{P}=\mathrm{A} \vee \mathrm{~B} \\
& \text { if } B=\text { true, } p \text { is always true. } \\
& \text { so if } B=\text { false, } A \text { determines } p \text {. } \\
& \text { if } A=\text { false, } B \text { determines } p .
\end{aligned}
$$

- Goal : Find tests for each clause when the clause determines the value of the predicate
- This is formalized in several criteria that have subtle, but very important, differences


## KAIST

## Active Clause Coverage

Active $c_{i}$ in $C_{p}$, choose minor clauses $c_{j} ; j!=i$, so that $c_{i}$ determines
clauser
$p$. TR has two requirements for each $c_{i}: c_{i}$ evaluates to true and $c_{i}$
evaluates to false.


- This is a form of MCDC, which is required by the Federal Avionics Admini stration (FAA) for safety critical software
- Ambiguity: Do the minor clauses have to have the same values when the major clause is true and false?


## KAIST

## Resolving the Ambiguity

$\quad \mathrm{p}=\mathrm{a} \vee(\mathrm{b} \wedge \mathrm{c})$
Major clause : a
$\mathrm{a}=$ true, $\mathrm{b}=$ false, $\mathrm{c}=$ true
$\mathrm{a}=$ false, $\mathrm{b}=$ false $\mathrm{c}=$ false

## Is this allowed?

- This question caused confusion among testers for years
- Considering this carefully leads to three separate criteria :
- Minor clauses do not need to be the same (GACC)
- Minor clauses do need to be the same (RACC)
- Minor clauses force the predicate to become both true and false (CACC)


## KAIST

## General Active Clause Coverage

Generan : For each $p$ in $P$ and
each major clause $c_{i}$ in $C p$, choose minor clauses $c_{j} j!=i$, so that
$c_{i}$ determines $p$. TR has two requirements for each $c_{i}: c_{i}$ evaluates
to true and $c_{i}$ evaluates to false.
The values chosen for the minor clauses $c_{j}$ do not need to be the
same when $c_{i}$ is true as when $c_{i}$ is false, that is, $c_{j}\left(c_{i}=\right.$ true $)=c_{j}\left(c_{i}=\right.$
false) for all $c_{j}$ OR $c_{j}\left(c_{i}=\right.$ true) $!=c_{j}\left(c_{i}=\right.$ false) for all $c_{j}$.

- It is possible to satisfy GACC without satisfying predicate coverage
- Ex. $\mathrm{p}=\mathrm{a} \leftrightarrow \mathrm{b}$,
\{TT, FF\} satisfies GACC, but not PC
- We want to cause predicates to be both true and false!


## Restricted Active Clause Coverage

> : For each $p$ in $P$ and each major clause $c_{i}$ in $C p$, choose minor clauses $c_{j 1} j!=i$, so that $c_{i}$ determines $p$. TR has two requirements for each $c_{i}: c_{i}$ evaluates to true and $c_{i}$ evaluates to false.

> The values chosen for the minor clauses $c_{j}$ must be the same when $c_{i}$ is true as when $c_{i}$ is false, that is, it is required that $c_{j}\left(c_{i}=\right.$ true $)=c_{j}\left(c_{i}=\right.$ false $)$ for all $c_{j}$.

- This has been a common interpretation by aviation developers
- RACC often leads to infeasible test requirements
- There is no logical reason for such a restriction


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## Correlated Active Clause Coverage

Correlan : For each $p$ in $P$ and
each major clause ci in $C p$, choose minor clauses $c_{j}, j!=i$, so that
$c_{i}$ determines $p$. TR has two requirements for each $c_{i}: c_{i}$ evaluates
to true and $c_{i}$ evaluates to false.
The values chosen for the minor clauses $c_{j}$ must cause $p$ to be
true for one value of the major clause $c_{i}$ and false for the other,
that is, it is required that $p\left(c_{i}=\right.$ true) $!=p\left(c_{i}=\right.$ false).
A more recent interpretation
Implicitly allows minor clauses to have different values
Explicitly satisfies (subsumes) predicate coverage

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## CACC and RACC

|  | a | b | c | $\mathrm{a} \wedge(\mathrm{b} \vee \mathrm{c})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | a |  | T | T |
| 1 | T | T | T |  |
| 2 | T | T | F | T |
| 3 | T | F | T | T |
| 5 | F | T | T | F |
| 6 | F | T | F | F |
| 7 | F | F | T | F |


|  |  | a | b | c |
| :---: | :---: | :---: | :---: | :---: |
|  | a | $\mathrm{a} \wedge(\mathrm{b} \vee \mathrm{c})$ |  |  |
| 1 | T | T | T | T |
| 5 | F | T | T | F |
| 2 | T | T | F | T |
| 6 | F | T | F | F |
| 3 | T | F | T | T |
| 7 | F | F | T | F |

major
clause

CACC can be satisfied by choosing any of rows 1, 2, 3
AND any of rows 5, 6, 7 - a total of nine pairs

## Inactive Clause Coverage

- The active clause coverage criteria ensure that "major" clauses do affect the predicates
- Inactive clause coverage takes the opposite approach - major clauses do not affect the predicates



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## General and Restricted ICC

- Unlike ACC, the notion of correlation is not relevant
- $c_{i}$ does not determine $p$, so cannot correlate with $p$
- Predicate coverage is always guaranteed




## KAIST

## Logic Coverage Criteria Subsumption



## Making Clauses Determine a Predicate

- Finding values for minor clauses $C_{j}$ is easy for simple predicates
- But how to find values for more complicated predicates?
- Definitional approach:
- $p_{c=t r u e}$ is predicate $p$ with every occurrence of $c$ replaced by true
- $p_{c=f a l s e}$ is predicate $p$ with every occurrence of $c$ replaced by false
- To find values for the minor clauses, connect $p_{C=\text { true }}$ and $p_{C=f a l s e}$ with exclusive $O R$

$$
p_{c}=p_{c=t r u e} \oplus p_{c=\text { false }}
$$

- After solving, $p_{C}$ describes exactly the values needed for $C$ to deter mine $p$


## Examples

```
            \(p=a \vee b\)
\(p_{a}=p_{a=\text { true }} \oplus p_{a=\text { false }}\)
    \(=(\) true \(\vee b)\) XOR (false \(\vee b)\)
    = true XOR b
    = \(\quad\) b
```

|  | $\quad \frac{p=a \wedge b}{}$ |
| ---: | :--- |
| $p_{a}$ | $=p_{a=t r u e} \oplus p_{a=\text { false }}$ |
|  | $=($ true $\wedge b) \oplus($ false $\wedge b)$ |
|  | $=b \oplus$ false |
|  | $=b$ |

    = b
    ```
p=a\vee(b\wedgec)
pa}=\mp@subsup{p}{a=true}{}\oplus\mp@subsup{p}{a=false}{
    = (true \vee (b ^c)) }\oplus(\mathrm{ false }\vee(b\wedgec)
    = true }\oplus(b\wedgec
    = ᄀ (b ^c)
    = ᄀ b \vee ᄀc
```

- "NOT b $\vee$ NOT $c$ " means either $b$ or $\boldsymbol{c}$ can be false
- RACC requires the same choice for both values of a, CACC KAIST does not


## A More Subtle Example

```
\(p_{\mathrm{a}}=\mathrm{p}_{\mathrm{a}=\text { true }} \oplus \mathrm{p}_{\mathrm{a}=\text { false }}\)
    \(=((\) true \(\wedge \mathbf{b}) \vee(\) true \(\wedge \neg \mathbf{b})) \oplus((\) false \(\wedge \mathbf{b}) \vee(\) false \(\wedge \neg \mathbf{b}))\)
    \(=(b \vee \neg b) \oplus\) false
    = true \(\oplus\) false
    = true
```

$$
p=(a \wedge b) \vee(a \wedge \neg b)
$$

$$
\mathrm{p}_{\mathrm{b}}=\mathrm{p}_{\mathrm{b}=\text { true }} \oplus \mathrm{p}_{\mathrm{b}=\text { false }}
$$

$$
=((a \wedge \text { true }) \vee(a \wedge \neg \text { true })) \oplus((a \wedge \text { false }) \vee(a \wedge \neg \text { false }))
$$

$$
=(a \vee \text { false }) \oplus(\text { false } \vee a)
$$

$$
=\mathbf{a} \oplus \mathbf{a}
$$

= false

- a always determines the value of this predicate
- $\boldsymbol{b}$ never determines the value - $\boldsymbol{b}$ is irrelevant !


## KAIST

## Infeasible Test Requirements

- Consider the predicate:

$$
(a>b \wedge b>c) \vee c>a
$$

- $(a>b)=$ true $(b>c)=$ true, $(c>a)=$ true is infeasible
- As with graph-based criteria, infeasible test requirements have to be recognized and ignored
- Recognizing infeasible test requirements is hard, and in general, undecidable


## Example

$$
p=a \wedge(\neg b \vee c)
$$

|  | a | b | c | p | $\mathrm{p}_{\mathrm{a}}$ | $\mathrm{p}_{\mathrm{b}}$ | $\mathrm{p}_{\mathrm{c}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | T | T | T | T | T | F | T |
| 2 | T | T | F | F | F | T | T |
| 3 | T | F | T | T | T | F | F |
| 4 | T | F | F | T | T | T | F |
| 5 | F | T | T | F | F | F | F |
| 6 | F | T | F | F | F | F | F |
| 7 | F | F | T | F | T | F | F |
| 8 | F | F | F | F | T | F | F |

- Conditions under which ear of the clauses determines $\gamma$
- $\mathrm{p}_{\mathrm{a}}:(\neg \mathrm{b} \vee \mathrm{c})$
- $p_{b}: a \wedge \neg C$
- $p_{c}: a \wedge b$


## KAIST

## Logic Coverage Summary

- Predicates are often very simple-in practice, most have less t han 3 clauses
- In fact, most predicates only have one clause!
- With only clause, PC is enough
- With 2 or 3 clauses, CoC is practical
- Advantages of ACC and ICC criteria significant for large predicates
- CoC is impractical for predicates with many clauses
- Control software often has many complicated predicates, with lots of clauses
- Question ... why don't complexity metrics count the number of clauses in predicates?

